Enhancement of the critical temperature of superconductors by Anderson localization

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Mesoscopic and strongly correlated electron systems – 6

Non-equilibrium and coherent phenomena at nanoscale

Chernogolovka, June 18, 2012

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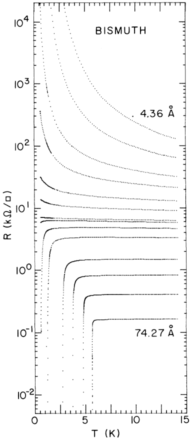
Published in

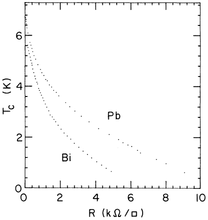
Phys. Rev. Lett. 108, 017002 (2012)

SIT in homogeneously disordered materials

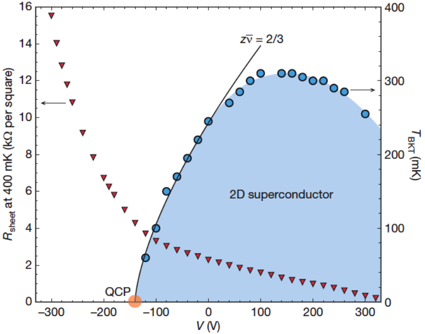
- amorphous Mo-Ge films (thickness $b = 15 - 1000$ Å)
  Graybeal, Beasley (1984)
- Bi and Pb layers on amorphous Ge ($b = 4 - 75$ Å)
  Strongin et al. (1971); Haviland, Liu, Goldman (1989)
- ultrathin Be films ($b = 4 - 15$ Å)
  Bielejec, Ruan, Wu (2001)
- amorphous thick In-O films ($b = 100 - 2000$ Å)
  Shahar, Ovadyahu (1992); Gantmakher (1998); Gantmakher et al. (1998),(2000); Sambandamurthy et al. (2004); Sacépé et al. (2011)
- thin TiN films
  Baturina et al. (2007)
- Li_xZrNCl powders
  Kasahara et al. (2009)
- $LaAlO_3/SrTiO_3$ interface
  Caviglia et al. (2008)

for recent review, see Gantmakher, Dolgopolov (2010)





Bi and Pb films on amorphous Ge layer
After Haviland, Liu, Goldman (1989)

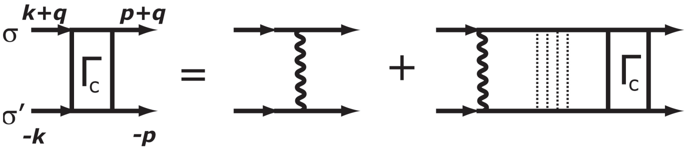


Caviglia et al. (2008)

Giant background dielectric constant: Coulomb interaction strongly screened

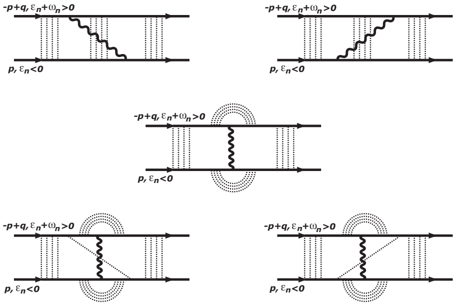
Nonmagnetic impurities do not affect s-wave superconductors:

Cooper-instability is the same for clean and diffusive electrons



Mean free path l does not enter expression for T_c

Disorder, Coulomb (long-ranged) repulsion,
(short-ranged) attraction in the Cooper channel



Diagrams for renormalization of attraction in the Cooper channel

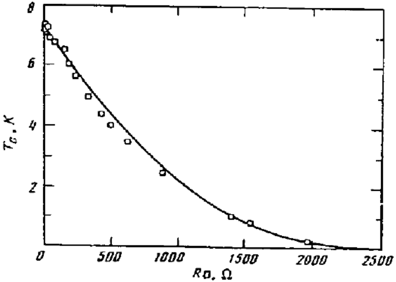
Suppression of T_c in a film as compared with BCS result

$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\Box \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^3 < 0$$

Ovchinnikov (1973) (wrong sign); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982); Finkelstein (1987)

RG theory for disorder and interactions

Finkelstein (1983); Castellani, Di Castro, Lee, Ma (1984)



after Finkelstein (1994) for Mo-Ge films

T_c vanishes at the sheet resistance

$$R_\square \sim \left(\ln \frac{1}{T_c^{BCS} \tau} \right)^{-2}$$

Finkelstein (1987)

BCS model for the exact electron states in a given disorder

Bulaevskii, Sadovskii (1984); Ma, Lee (1985); Kapitulnik, Kotliar (1985)

superconductivity with $T_c^{BCS} \propto \omega_D \exp(-2/\lambda)$ survives as long as

$$T_c^{BCS} \gtrsim \delta_\xi \propto 1/(\nu_d \xi^d)$$

where ξ – localization length, ν_d – density of state

Enhancement of T_c as compared with BCS results

$$T_c \propto \tau^{-1} \lambda^{d/|\Delta_2|}$$

Feigelman et al. (2007, 2010)

$\Delta_2 < 0$ – multifractal exponent for inverse participation ratio

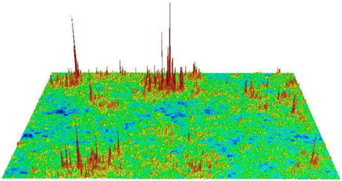
Multifractality near Anderson transition (No e-e interactions)

Wegner (1980); Kravtsov, Lerner (1985); Pruisken(1985); Castellani, Peliti (1986)

$$\left\langle \int d^d\boldsymbol{r}\, |\phi_\varepsilon(\boldsymbol{r})|^{2q} \right\rangle \sim L^{-\tau_q}$$

perfect metal: $\tau_q = d(q-1)$ – perfect insulator: $\tau_q = 0$

criticality: $\tau_q = d(q-1) + \Delta_q$



from Evers, Mildenberger and Mirlin

if $f(\alpha)$ is Legendre transform of τ_q: $f(\alpha) = q\alpha - \tau_q, \quad \alpha = d\tau_q/dq$

then $L^{f(\alpha)}$ measures a set of points where $|\phi_\varepsilon|^2 \sim L^{-\alpha}$

Can suppression of T_c due to Coulomb repulsion and enhancement of T_c due to multifractality be described in a unified way ?

Does weak multifractality enhances T_c in 2D systems ?

Does the enhancement of T_c hold if one takes into account short-ranged repulsion in particle-hole channels ?

Free electrons:

$$H_0 = \int d^d\boldsymbol{r}\,\bar{\psi}_\sigma(\boldsymbol{r})\left[-\frac{\nabla^2}{2m}\right]\psi_\sigma(\boldsymbol{r})$$

where $\sigma = \pm 1$ is spin projection

Scattering off white-noise random potential :

$$H_{\rm dis} = \int d^d\boldsymbol{r}\,\bar{\psi}_\sigma(\boldsymbol{r})\,V(\boldsymbol{r})\psi_\sigma(\boldsymbol{r})$$

Gaussian distribution: $\langle V(\boldsymbol{r})\rangle = 0$, $\langle V(\boldsymbol{r})\,V(0)\rangle = (2\pi\nu_d\tau)^{-1}\delta(\boldsymbol{r})$

Electron-electron interaction:

$$H_{\rm int} = \frac{1}{2}\int d^d\boldsymbol{r_1}\, d^d\boldsymbol{r_2}\, \bar{\psi}_\sigma(\boldsymbol{r_1})\psi_\sigma(\boldsymbol{r_1})\, U(\boldsymbol{r_1}-\boldsymbol{r_2})\, \bar{\psi}_{\sigma'}(\boldsymbol{r_2})\psi_{\sigma'}(\boldsymbol{r_2})$$

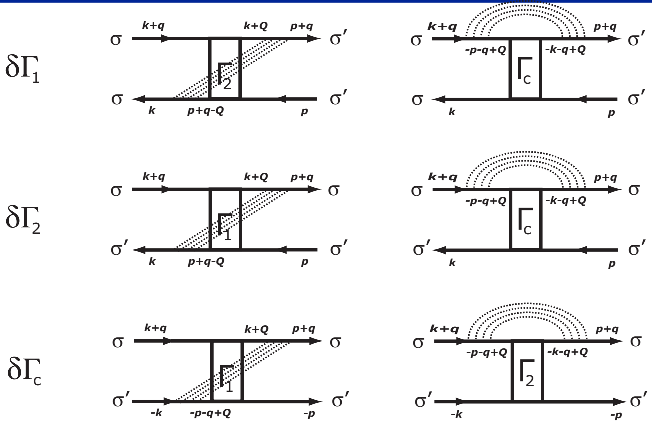
for the case of short-ranged repulsion with BCS-type attraction

$$U(\boldsymbol{R}) = u_0\left[1+\frac{R^2}{a^2}\right]^{-\alpha/2} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R}), \quad \alpha > d, \quad u_0 > 0, \quad \lambda > 0$$

for the case of Coulomb repulsion with BCS-type attraction

$$U(\boldsymbol{R}) = \frac{e^2}{\epsilon R} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R})$$

Renormalization of interaction parameters / first order in interaction



Maekawa, Fukuyama (1982); Takagi, Kuroda (1982); Castellani et al. (1984); Finkelstein (1987)

$$\gamma_1 \equiv (\gamma_t - \gamma_s)/2 \text{ and } \gamma_2 \equiv \gamma_t$$

$$\frac{dt}{dy} = t^2\left[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c\right]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}\left[1 + \gamma_s\right]\left[\gamma_s + 3\gamma_t + 2\gamma_c\right]$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}\left[1 + \gamma_t\right]\left[\gamma_s - \gamma_t - 2\gamma_c(1 + 2\gamma_t)\right]$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[\gamma_s - 3\gamma_t + \gamma_c(\gamma_s + 3\gamma_t)\right] - 2\gamma_c^2$$

Finkelstein (1984, 1987); Castellani et al. (1984), Ma, Fradkin (1986)

where $y = \ln L/l$ and $f(x) = 1 - (1 + x^{-1})\ln(1 + x)$

lowest order in disorder, $t = 2/\pi g$, g is conductivity in units e^2/h

exact in γ_s (singlet p-h channel) and γ_t (triplet p-h channel)

lowest order in γ_c (cooper channel)

Coulomb (long-ranged) interaction: $\gamma_s = -1$

$$\frac{dt}{dy} = t^2\left[1 + 1 + 3f(\gamma_t) - \gamma_c\right]$$
$$\frac{d\gamma_t}{dy} = \frac{t}{2}\left[1 + \gamma_t\right]\left[1 + \gamma_t + 2\gamma_c(1 + 2\gamma_t)\right]$$

$$\frac{d\gamma_c}{dy} = \frac{t}{2}\left[1 + 3\gamma_t\right] - 2\gamma_c^2 \qquad \Longrightarrow \qquad \gamma_c^2 \sim t(1 + 3\gamma_t) > 0$$

destruction of superconductivity by disorder and Coulomb interaction

Finkelstein (1987)

$$\frac{dt}{dy} = t^2\Big(1 - [\gamma_s + 3\gamma_t + 2\gamma_c]/2\Big)$$

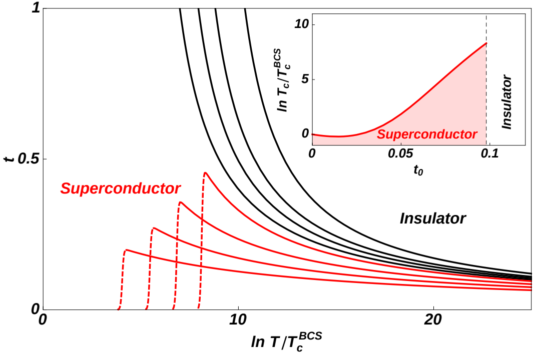
$$\frac{d}{dy}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 3 & 2\\ 1 & -1 & -2\\ 1 & -3 & 0\end{pmatrix}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} - \begin{pmatrix}0\\ 0\\ 2\gamma_c^2\end{pmatrix}$$

weak interaction, $|\gamma_s|, |\gamma_t|, |\gamma_c| \ll 1$

weak disorder, $t \ll 1$.

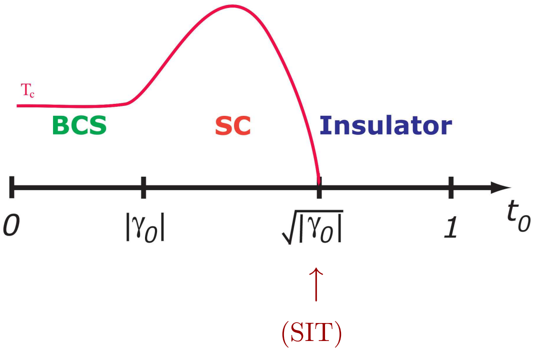
initial values $\gamma_s(0) < 0$, $\gamma_t(0) > 0$, $\gamma_c(0) = \gamma_{c0} < 0$, $t(0) = t_0$

for $|\gamma_{c0}| \ll t_0 \ll 1$ attraction to the (BCS) line $-\gamma_s = \gamma_t = \gamma_c \equiv \gamma$



$\gamma_{s0} = -0.005$, $\gamma_{t0} = 0.005$, $\gamma_{c0} = -0.04$,

$t_0 = 0.065, 0.075, 0.085, 0.095, 0.10, 0.105, 0.11, 0.12$ (from bottom to top) sketch of phase diagram



enhancement of T_c due to weak multifractality: $T_c \sim \tau^{-1} e^{-2y_\star} \approx \tau^{-1} e^{-2/t_0} \gg T_c^{BCS}$

RG equation for the effective attraction

$$\frac{d\gamma}{dy} = 2t\gamma - 2\gamma^2/3$$

step 1 for $t > \gamma$: neglect Cooper instability, enhancement of interaction matrix element due to weak multifractality

step 2 for $\gamma > t$: neglect multifractality then conventional BCS, but with the new “bare” coupling constant t determined by disorder

enhancement of the mean-field T_c

Role of perpendicular magnetic field $H_\perp$ in 2D / for $|\gamma_0| \ll t_0 \ll \sqrt{|\gamma_0|}$

$H_\perp$ suppresses Cooper channel interaction at $L > l_H = \sqrt{1/eH_\perp}$

for $l_H > le^{y_*}$, SC survives
for $l_H < le^{y_*}$, SC instability is blocked

at $y > \ln l_H/l$ the system flows towards insulator according to

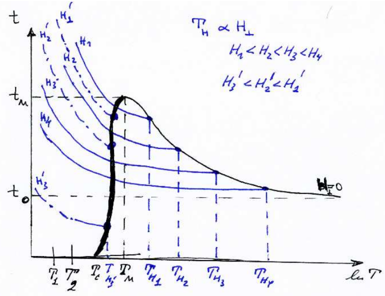
$$\frac{dt}{dy} = t^2\Big[f(\gamma_s) + 3f(\gamma_t)\Big]$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}\big[1+\gamma_s\big]\big[\gamma_s + 3\gamma_t\big]$$
$$\frac{d\gamma_t}{dy} = -\frac{t}{2}\big[1+\gamma_t\big]\big[\gamma_s - \gamma_t\big]$$

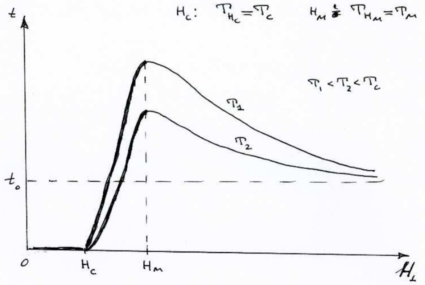
Finkelstein (1984); Castellani et al. (1984)

with unusual initial conditions $\gamma_{t0} = \gamma_c(\ln l_H/l) < 0$ and $\gamma_{s0} = -\gamma_c(\ln l_H/l) > 0$

SC is destroyed by $H_\perp$ at $l_H < le^{y_*}$ or, equivalently, $\mu_B H_\perp > T_c$

Role of perpendicular magnetic field $H_\perp$ in 2D / sketch of magnetoresistance





there is Coulomb (rather than short-ranged) repulsion in experiments with SIT in 2D

in case of large background dielectric constant the static screening length $\varkappa^{-1} \gg l$.

our results for T_c in 2D are valid for Coulomb interaction (with $\varkappa l \ll 1$) provided

$$\ln 1/(\varkappa l) \gg \min\{|\gamma_{c0}|^{-1}, t_0^{-1}\}$$

RG equations near free electron fixed point $t = t_c$, $\gamma = 0$:

$$\frac{dt}{dy} = \frac{1}{\nu}(t - t_c) + \eta\gamma, \qquad \frac{d\gamma}{dy} = -\Delta_2\gamma - a\gamma^2\,, \quad a \sim 1$$

Initial values: $t(0) = t_0$ and $\gamma(0) = \gamma_0 < 0$

Correlation length: $\xi = \left|\tilde{t}_0 - t_c\right|^{-\nu}$

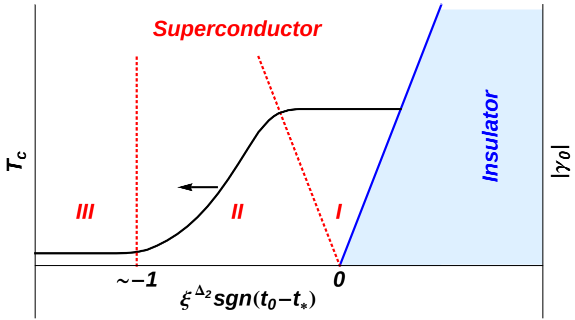
where $\tilde{t} = t - \frac{\eta\nu\gamma}{|\Delta_2|\nu - 1}$ and $\tilde{t}_0 = \tilde{t}(0)$

transform from t to $\tilde{t}$ removes $\eta\gamma$ from the first equation

3D Anderson transition (orth. sym. class): $\nu = 1.57 \pm 0.02$ and $\Delta_2 = -1.7 \pm 0.05$

for a review see e.g., Mirlin, Evers (2010)

Schematic diagram in the interaction–disorder plane and T_c



III: $T_c = T_c^{BCS}$ II: $T_c = \xi^{-d} E_0 \exp\left(-\frac{d}{a|\gamma_0|\xi^{|\Delta_2|}}\right)$ I: $T_c = E_0 |\gamma_0|^{d/|\Delta_2|}$

T_c for region I coincides with Feigelman et al. (2007)

Conclusions

enhancement of T_c in 2D due to weak multifractality at $|\gamma_{c0}| \ll t_0 \ll \sqrt{|\gamma_{c0}|} \ll 1$

- weak short-ranged repulsion does not prevent from SC
- non-monotonous magnetoresistane due to non-monotonous T-dependence of resistivity at zero field
- in case $\ln(1/\varkappa l) \gg 1/t_0$ there is an enhancement of T_c in spite of Coulomb repulsion

strong enhancement of T_c near (free electron) Anderson transition even in the presence of weak short-ranged repulsion