

Best regards from Norway.  
Thanks for the invitation.



*Saltdalsfjorden - Norway*

# Low-Frequency Noise as a Source of Non-Gaussian Decoherence in Qubits



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Discussions: Y. Nakamura, J. S. Tsai, Y. A. Pashkin, T. Yamamoto, O. Astafiev

## Support:



 Norges forskningsråd



# Outline

- Motivation
- Reminder: Qubits, single Cooper pair box, etc
- Dynamical defects (fluctuators) as a source of flicker-noise
- Decoherence in driven systems
- Non-Gaussian decoherence by a single fluctuator
- Effect of many fluctuators
- Conclusions
- Relation between energy relaxation and decoherence
- Decoherence close to optimal point
- Resonant interaction between qubit and fluctuators (talk by A. Ustinov)
- Role of non-stationary fluctuations



Lara Faoro

THE STATE UNIVERSITY OF NEW JERSEY  
**RUTGERS**

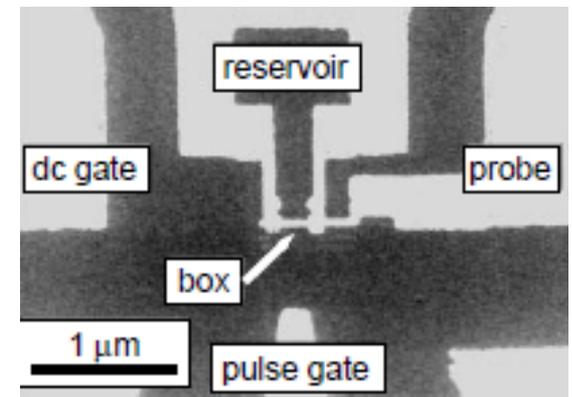


Ivar Martin

 **Los Alamos**  
NATIONAL LABORATORY  
EST. 1943

## Motivation:

Q: In which way low frequency noise destroys quantum coherence?



Charge qubit

VOLUME 88, NUMBER 4

PHYSICAL REVIEW LETTERS

28 JANUARY 2002

### Charge Echo in a Cooper-Pair Box

Y. Nakamura,<sup>1</sup> Yu. A. Pashkin,<sup>2,\*</sup> T. Yamamoto,<sup>1</sup> and J. S. Tsai<sup>1</sup>

A spin-echo-type technique is applied to an artificial two-level system that utilizes a charge degree of freedom in a small superconducting electrode.

.....

Comparison of the decay time of the observed echo signal with an estimated decoherence time suggests that low-frequency energy-level fluctuations due to the  $1/f$  charge noise dominate the dephasing in the system.

# What is a qubit?



Quantum two-level system equivalent to  $\frac{1}{2}$  spin

The qubit is described by effective Hamiltonian

$$\mathcal{H}_{\text{ctrl}} = -\frac{1}{2}B_z \hat{\sigma}_z - \frac{1}{2}B_x \hat{\sigma}_x .$$

$\hat{\sigma}_i$  - Pauli matrices

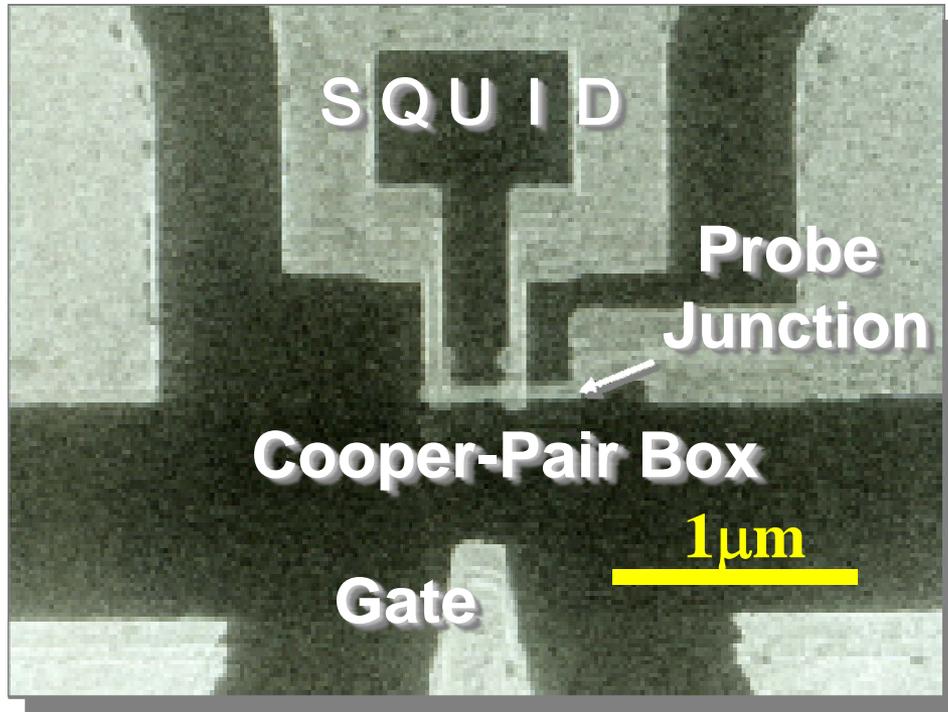
with tunable  $B_x$  and  $B_z$  to perform **single-qubit** operations.

A controllable interaction in the form

$$\mathcal{H}_{\text{ctrl}}(t) = -\frac{1}{2} \sum_{i=1}^N B^i(t) \hat{\sigma}^i + \sum_{i \neq j} J_{ab}^{ij}(t) \hat{\sigma}_a^i \hat{\sigma}_b^j ,$$

(where a summation over spin indices  $a, b = x, y, z$  is implied) to perform **two-bit** operations.

# Josephson Charge Qubit: Reminder

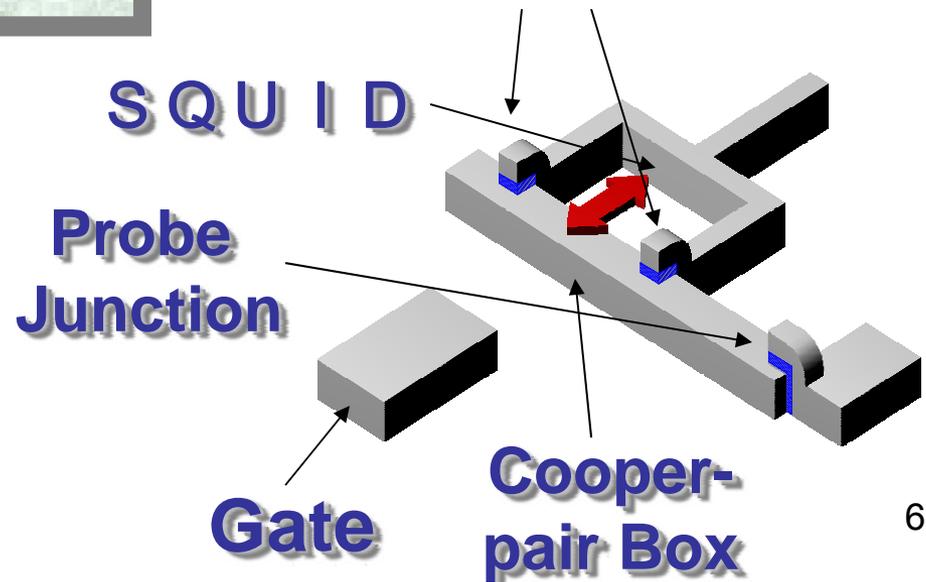


Artificial  $\frac{1}{2}$ -spin -  
Josephson qubit  
(NEC, Japan)

Nature 2003, 2004

Europhysics Prize, 2004

Al/AlO<sub>x</sub>/Al tunnel junctions



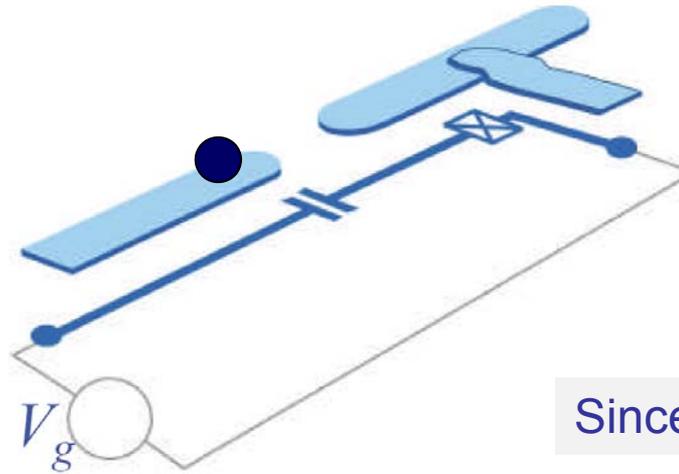
Why this system is a  
controllable  $\frac{1}{2}$  spin?

# (Normal) single-electron box

How much we pay to transfer  $N$  electrons?

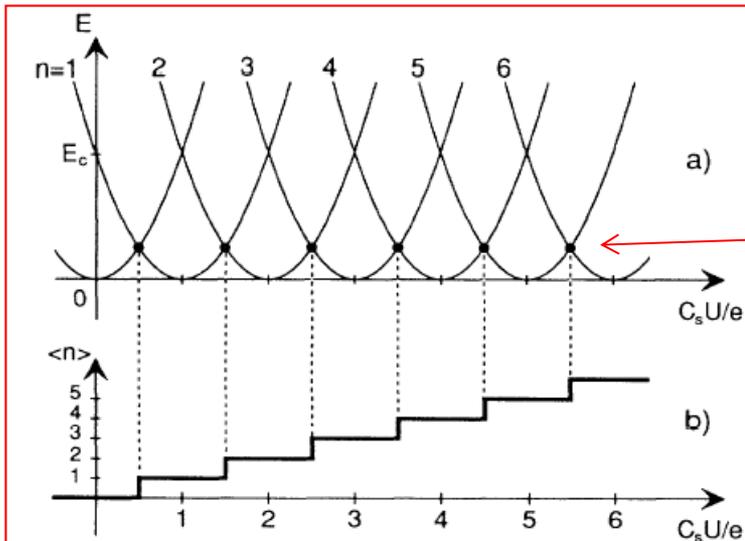
$$E = \frac{Q^2}{2C} + QV_g \quad E_{\min} = -\frac{CV_g^2}{2}$$

$$E - E_{\min} = \frac{(Q + CV_g)^2}{2C}$$



Since the electron charge is discrete,  $Q = -eN \rightarrow$

$$E - E_{\min} = E_c(N - \alpha V_g)^2, \quad E_c = \frac{e^2}{2C}, \quad \alpha = \frac{C}{e}$$



At some values of the gate voltage the electron transfer is free of energy cost!

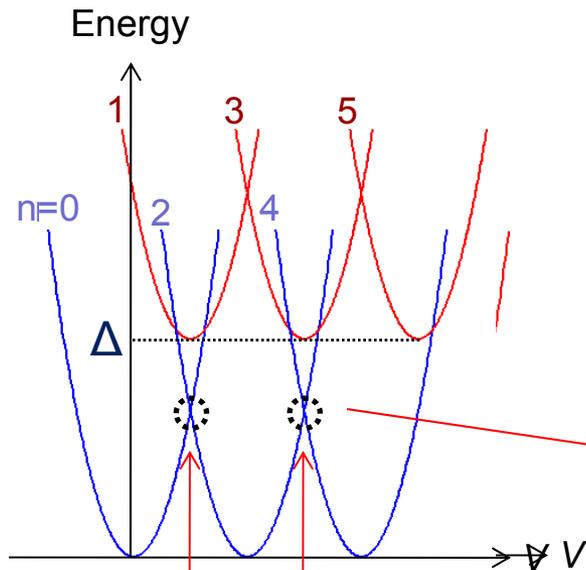
Otherwise the transfer is exponentially suppressed (Coulomb blockade)

In a superconductor, energy depends on the **parity** of the electron number

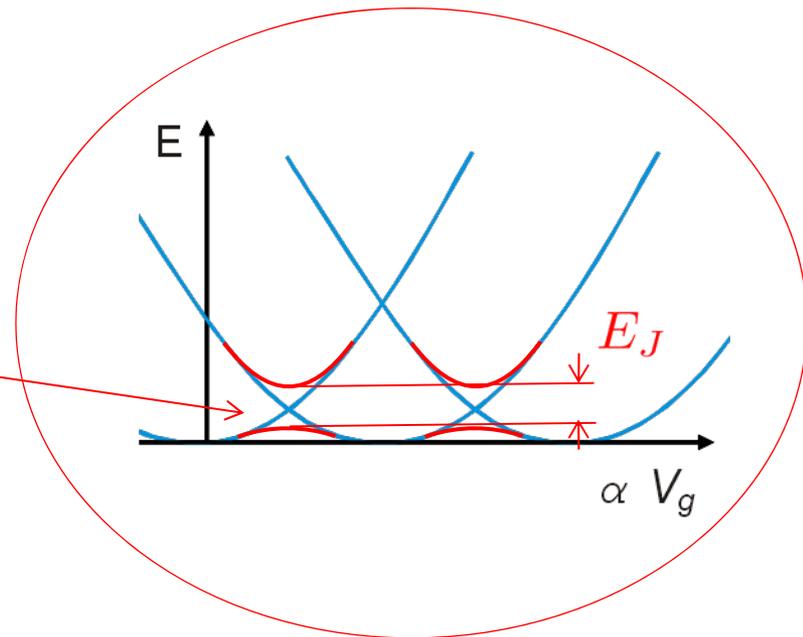
$$E(N) = E_C(N - \alpha V_g)^2 + \Delta_N$$

$$\Delta_N = \begin{cases} 0, & N = 2n \\ \Delta, & N = 2n + 1 \end{cases}$$

$$E_C = (2e)^2/2C, \quad \Delta - \text{gap}$$



Cost free transfer of  
**Cooper pairs**



The splitting is due to the Josephson tunneling of Cooper pairs:

$$\mathcal{H}_J = -\frac{E_J}{2} (|N\rangle\langle N+1| + (|N+1\rangle\langle N|))$$

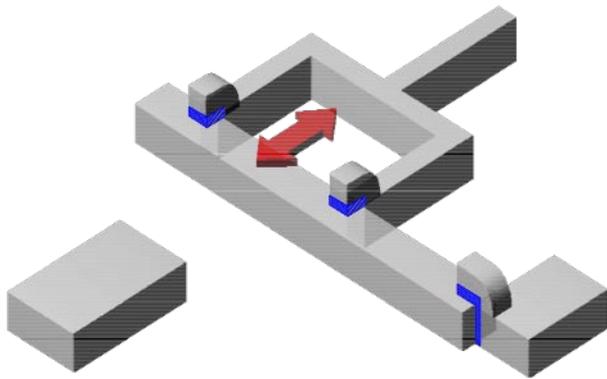
## Effective Hamiltonian (in the space of excess Cooper pairs number):

$$\tilde{\mathcal{H}} = -\frac{1}{2} \underbrace{E_c(1 - \alpha V_g)}_{B_z} \sigma_z - \frac{1}{2} \underbrace{E_J}_{B_x} \sigma_x$$

$B_z$  can be tuned by the gate voltage. How can we tune the effective field  $B_x$ ?

The Josephson junction is split into an interferometer. The energy is tuned by the magnetic flux in the SQUID loop, which modulates  $E_J$ .

$$E_J^{\text{eff}} = 2E_J \cos\left(\pi \frac{\Phi}{\Phi_0} + \varphi\right)$$



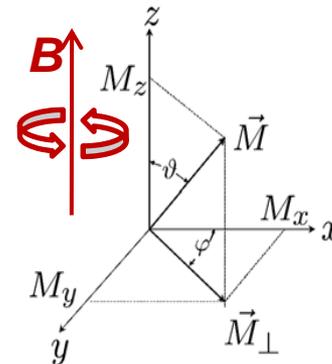
Single Cooper Pair Box

### Mapping on the Bloch sphere

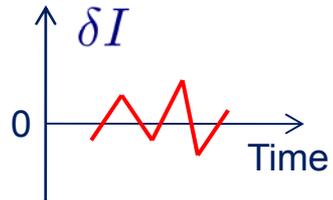
density matrix:  $\hat{\rho} = (\hat{I} + \mathbf{M} \cdot \hat{\sigma}) / 2$

Hamiltonian:  $\mathcal{H}_q = \mathbf{B} \cdot \boldsymbol{\sigma} / 2$

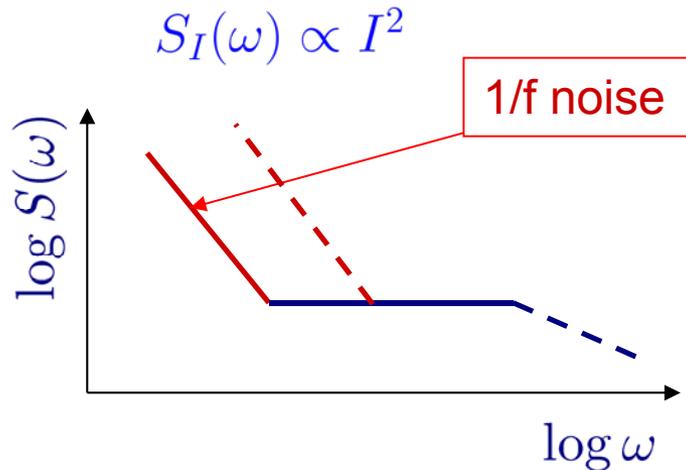
Eq. of motion:  $\dot{\mathbf{M}} = \mathbf{B} \times \mathbf{M}$



# Noise (main properties)



Noise spectrum:  $S_I(\omega) \equiv \int_0^\infty d\tau e^{i\omega\tau} \langle [\delta\hat{I}(t + \tau), \delta\hat{I}(t)]_+ \rangle$



For fluctuations in electrical current – noise of resistance

1/f noise is a generic phenomenon observed in numerous systems

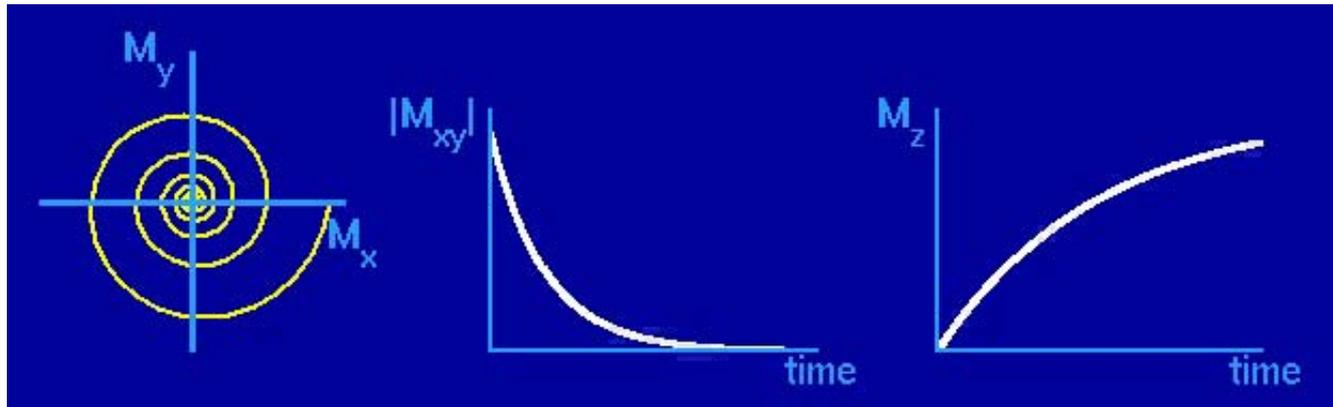
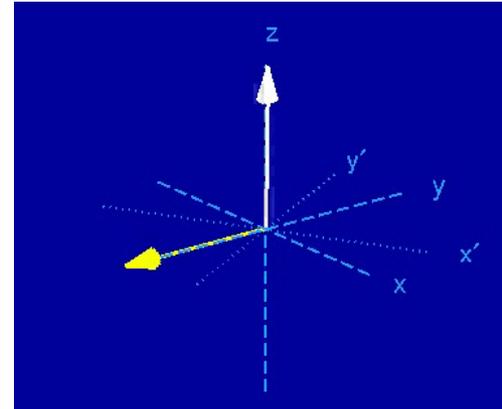
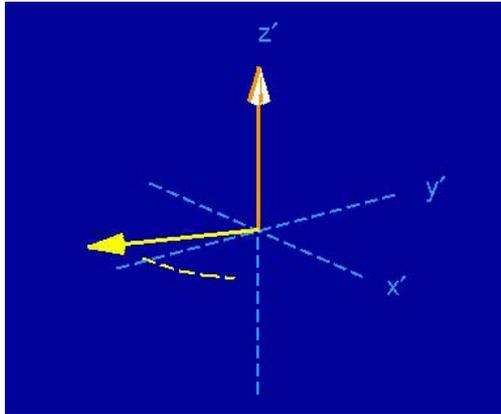
It is a non-equilibrium phenomenon, which is not described by fluctuation-to-dissipation theorem

Spectrum is rather universal – there is no saturation even at very low frequencies (down to  $10^{-7}$  Hz)

Q: In which way low frequency noise destroys quantum coherence?

# Noise-induced decoherence

Free induction protocol:



# Noise-induced decoherence

Environment induces stochastic noise:  $B = B_0 + b(t)$ ,  $b \parallel B_0$

Let  $b(t)$  be a classical Gaussian random variable.

Fluctuations of  $V_g$

Free induction signal:

$$m_+ \equiv \frac{M_x + iM_y}{\sqrt{M_x^2 + M_y^2}} \rightarrow m_+(t) = e^{i \int_0^t B(t') dt'} m_+(0)$$

$$\phi(t) = B_0 t + \int_0^t b(t') dt' = \phi_0(t) + \varphi(t)$$

Average:  $\langle m_+(t) \rangle = m_+(0) e^{i\phi_0(t)} \langle e^{i\varphi(t)} \rangle$

Though the length  $|\mathbf{M}|$  is conserved, the average  $|\langle \mathbf{M} \rangle|$  decays in time –  
decoherence

Averaging:

$$p(\varphi) = \frac{1}{\sqrt{2\pi\langle\varphi^2\rangle}} e^{-\varphi^2/2\langle\varphi^2\rangle}, \quad \langle\varphi^2\rangle = 4 \int_{-\infty}^{\infty} d\omega \frac{\sin^2 \frac{\omega t}{2}}{\omega^2} S(\omega)$$

$$S(\omega) = \frac{1}{\pi} \int_0^{\infty} dt \langle b(t)b(0) \rangle \cos \omega t$$

Noise spectrum

Correlator of random “magnetic fields”

Since  $\lim_{a \rightarrow \infty} \frac{\sin^2 ax}{\pi ax^2} = \delta(x)$  we get

$$\langle e^{i\varphi(t)} \rangle = e^{-\langle\varphi^2(t)\rangle/2} = e^{-t/T_2}, \quad T_2^{-1} = \pi S(0)$$

What about 1/f noise?



exponential decay of the signal at large times, the decrement being given by the **noise power at zero frequency**

# Quantum theory: Spin-Boson Model (sketch)

A.J. Leggett et al, *Rev. Mod. Phys.* v.59, 1 (1987).

U. Weiss, "Quantum Dissipative Systems", 2nd ed., (World Scientific, Singapore, 1999).

A. Shnirman, Y. Makhlin, and G. Schon, *Physica Scripta* v.T102, 147 (2002)

D. Loss and D. DiVincenzo, *cond-mat/030411*

1/2-spin linearly coupled to a bath consisting of oscillators:

$$\mathcal{H}_{s-b} = \sigma_z \hat{\mathcal{X}}, \quad \hat{\mathcal{X}} = \sum_j C_j (\hat{b}_j + \hat{b}_j^\dagger)$$

Decoherence is expressed through bath noise spectrum:

$$S_{\mathcal{X}}(\omega) \equiv \left\langle \left[ \hat{\mathcal{X}}(t), \hat{\mathcal{X}}(0) \right]_+ \right\rangle_{\omega} = 2J(\omega) \coth \frac{\hbar\omega}{2T}$$
$$J(\omega) \equiv \pi \sum_j C_j \delta(\omega - \omega_j) \quad \text{- bath spectral density}$$

Result:

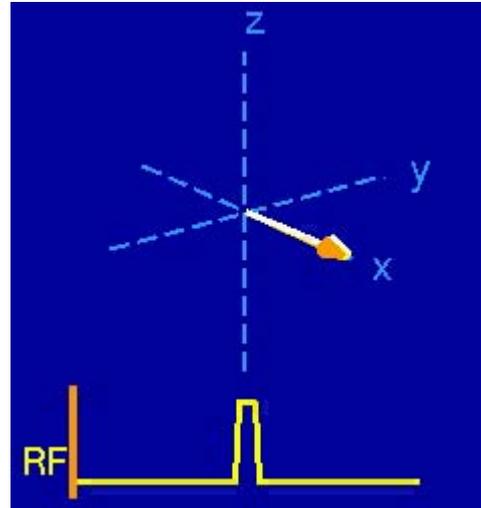
$$\langle m_+(t) \rangle \propto e^{-\mathcal{K}(t)} \quad \text{where} \quad \text{Re } \mathcal{K}(t) = \frac{4}{\pi} \int_0^\infty \frac{d\omega S(\omega)}{\omega^2} \sin^2 \frac{\omega t}{2}$$

The noise is assumed to be Gaussian

Everything is determined by the power spectrum of the noise !!!

However, for **1/f noise** the above integral is **divergent**.

# Spin-echo protocol



The *spin echo amplitude* can be expressed as  $\mathcal{P}(t) = e^{-\mathcal{K}_e(t)}$

where

$$\text{Re } \mathcal{K}(t) = 32 \int_0^{\infty} d\omega S_x(\omega) \frac{\sin^4(\omega t_{12}/2)}{\omega^2}$$

## Questions:

- The result is expressed only through the power spectrum (i.e., pair correlator) of the forces. This is perfect for Gaussian processes.

**Are non-Gaussian effects important?**

- For low-frequency noise with  $1/f$  spectrum the integral over frequencies is divergent, and it should be cut off

A. Shnirman, Y. Makhlin, and G. Schön, *Physica Scripta* **T102**, 147 (2002).

**Which cut-off procedure is appropriate?**

Our aim is to address these issue using a simple solvable model, which can be called **spin-fluctuator** model.

We will mostly discuss echo experiments since echo allows to find true dephasing.

## History:

Electronic emission in thermionic tube - Johnson (1925)

The name "flicker noise" belongs to Schottky (1926)

Carbon microphones - Christensen & Pearson (1936)

Various semiconductor devices - 1940-1950

"Explanation": The concept of non-exponential kinetics:

Assuming  $\langle \delta I(t) \delta I(0) \rangle = \overline{(\delta I)^2} e^{-|t|/\tau}$  one obtains:

$$S_I(\omega) = 2 \int_0^{\infty} \langle \delta I(t) \delta I(0) \rangle \cos(\omega t) dt = \overline{(\delta I)^2} \frac{2\tau}{1 + \omega^2 \tau^2}$$

In more general case,  $\delta I(t)$  is a superposition of many processes characterized by different relaxation times.

One can introduce a weight function,  $p_I(\tau)$ , including the number of the systems with given relaxation time

$$S_I(\omega) = \int_0^{\infty} d\tau p_I(\tau) \frac{2\tau}{1 + \omega^2 \tau^2} \quad \text{Surdin 1939}$$

$$p_I(\tau) \propto 1/\tau, \quad \tau_1 \ll \tau \ll \tau_2$$

$$S_I(\omega) \propto 1/\omega, \quad \tau_2^{-1} \ll \omega \ll \tau_1^{-1}$$

Distribution  $p_I(\tau) \propto \tau^{-1}$  is rather common, this is the case if  $\tau \propto e^W$  and  $W$  is smoothly distributed (tunneling, activation).

## Plan

- identify **fluctuators** – entities with exponentially broad distribution of relaxation rates
- using a concrete protocol consider interaction of the fluctuators with a qubit and estimate the decoherence
- estimate both spectrum and intensity of the noise produced by the fluctuators
- relate the decoherence and the noise and answer the question whether the decoherence is fully determined by the noise

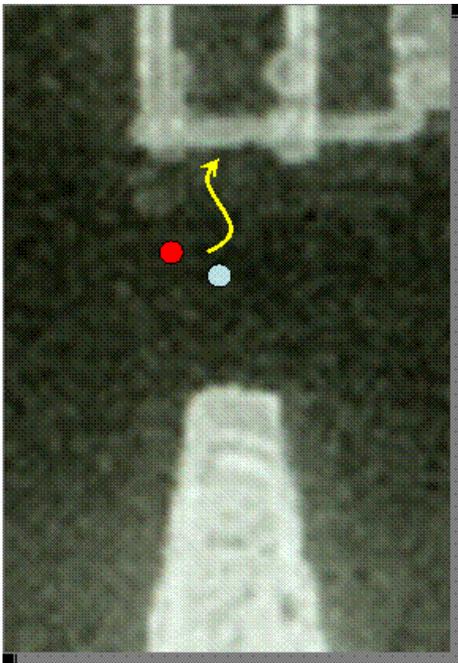
# Decoherence and energy relaxation: Spin-Fluctuator Model

**Fluctuators:** structural defects, charge traps, which can exist in dielectric parts of the device

The fluctuators randomly switch between their states due to interaction with extended modes of environment – phonons or electrons.

Switching  $\Rightarrow$  random fields  $\Rightarrow$  decoherence

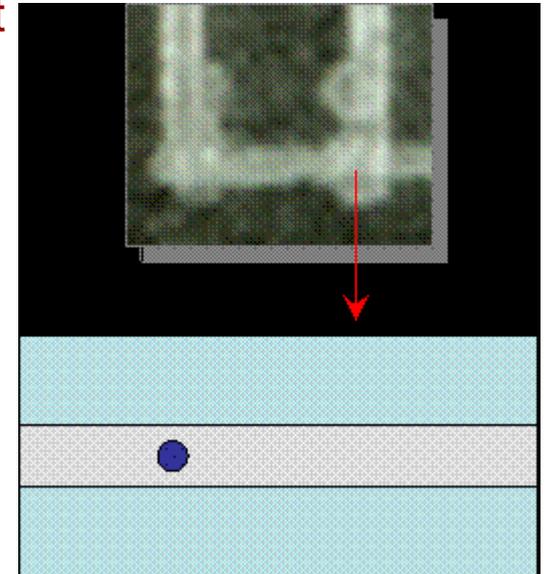
Modulation of induced charge



$$-\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x$$

↑                      ↑

Modulation of critical Josephson current



# Fluctuators during last 30 years

## Pioneering ideas:

**TLS in amorphous media:** P.W. Anderson, B. I. Halperin, and C. M. Varma, *Phil. Mag.* **25**, 1 (1972)  
W. A. Phillips, *J. Low Temp. Phys.* **7**, 351 (1972)

**Application to charge noise:** A. Ludviksson, R. Kree, A. Schmid, *PRL* **52**, 950 (1984)  
Sh.M. Kogan, K.E. Nagaev, *Sol. St. Comm.*, **49**, 387 (1984)

## Extensive work in between:

**Fluctuator-induced noise in point contacts and Josephson junctions:** V.I. Kozub, 1984 (several papers);  
C. Rogers and R. Buhrman, *PRL* **53**, 1272 (1984) + other experiments  
YMG, V.G. Karpov, V.I.Kozub, 1989;  
YMG, V.I. Gurevich, V.I. Kozub, 1989  
+ other theoretical and experimental activities

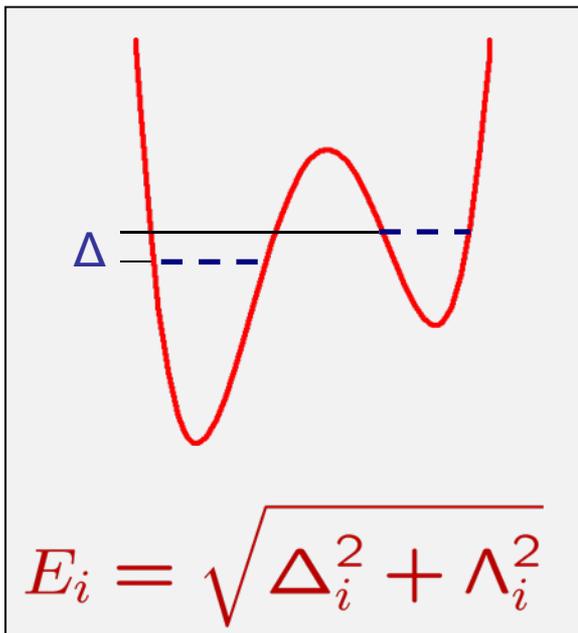
## Application to qubits:

**Charge noise:** Paladino, Faoro, Falci, and Fazio, *PRL* **88**, 228304 (2002); G. Falci *et al.* (2003-2005); Itakura & Tokura (2003); Schrieffer *et al.* (2006)

**Fluctuations of  $J_c$ :** Martinis, Nam, Aumentado, Lang, and Urbina, *PRB* **67**, 94510 (2003)

Hamiltonian:

$$\begin{aligned} \mathcal{H} &= -(B/2) \sigma_z + (1/2) F(t) \sigma_x && \text{qubit} \\ \text{fluctuator} &+ (1/2) \sum E_i \tau_z^{(i)} + \mathcal{H}_{\text{env}} + \mathcal{H}_{F-\text{env}} \\ \text{interaction} &+ \sum_i \left( v_i \sigma_z^{(i)} \tau_z^{(i)} + \dots \right) \end{aligned}$$



$$v_i = g(r_i) A(\mathbf{n}_i) (B_z/B) (\Delta_i/E_i)$$

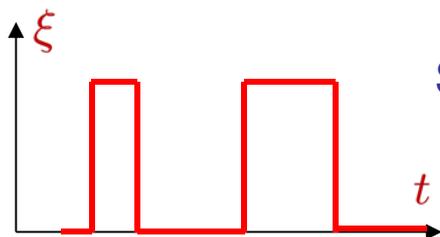
$$\begin{aligned} \mathcal{H}_{\text{env}} &= \sum_{\mu} \omega_{\mu} \left( \hat{b}^{\dagger} \hat{b} + \frac{1}{2} \right) \\ \mathcal{H}_{F-\text{env}} &= \sum_{i\mu} C_{i\mu} \tau_x^{(i)} (\hat{b}_{\mu} + \hat{b}_{\mu}^{\dagger}) \end{aligned}$$

## Simple classical model:

Classical low-frequency fluctuations  $\xi_i(t)$  acting upon the qubit:

$$\mathcal{H}_{qF} = \mathcal{X}_1(t) \sigma_z, \quad \mathcal{X}_1(t) = \sum_i v_i \xi_i(t)$$

Uncorrelated random telegraph processes:  $\xi_i(t) = 0$  or  $\xi_i(t) = 1$ .



Switching times are distributed according to Poisson distribution

$$\langle \xi_i(t) \xi_k(t') \rangle = \delta_{ik} e^{-2\gamma_i |t-t'|}$$

The switching rates,  $\gamma$ , are calculated in the 2<sup>nd</sup> order in the interaction between the fluctuator and the thermal bath:

$$\gamma_i = (1/2) \gamma_0(T) (\Lambda_i / E_i)^2$$

$$\Lambda \propto e^{-W} \rightarrow \mathcal{P}(\gamma) \propto \gamma^{-1}$$

Q: Does ensemble of fluctuators produce 1/f noise?

$$\begin{aligned} S_{\mathcal{X}}(\omega) &= 2 \int_0^\infty dt e^{i\omega t} \langle \mathcal{X}(t) \mathcal{X}(0) \rangle \\ &= 2 \sum_i v_i^2 \int_0^\infty dt e^{i\omega t} \langle \xi_i(t) \xi_i(0) \rangle \\ &\propto \sum_i v_i^2 \frac{\gamma_i}{\omega^2 + \gamma_i^2} \rightarrow \langle v^2 \rangle \int \mathcal{P}(\gamma) d\gamma \frac{\gamma}{\omega^2 + \gamma^2} \end{aligned}$$

For an **exponentially-broad** distribution of relaxation rates,  $\mathcal{P}(\gamma) \propto 1/\gamma$ ,  
and  $S_{\mathcal{X}}(\omega) \propto 1/\omega$ .

Interplay between decoherence and noise spectrum can depend on actual distribution of fluctuators in the device

# Is it possible to reduce decoherence by driving fluctuators?

Idea: M. Constantin, C. C. Yu, and J. M. Martinis, Phys. Rev. B **79**, 94520 (2009).

PHYSICAL REVIEW B

VOLUME 14, NUMBER 4

15 AUGUST 1976

## Intrinsic decay lengths of quasimonochromatic phonons in a glass below 1 K

Brage Golding, John E. Graebner, and R. J. Schutz

*Bell Laboratories, Murray Hill, New Jersey 07974*

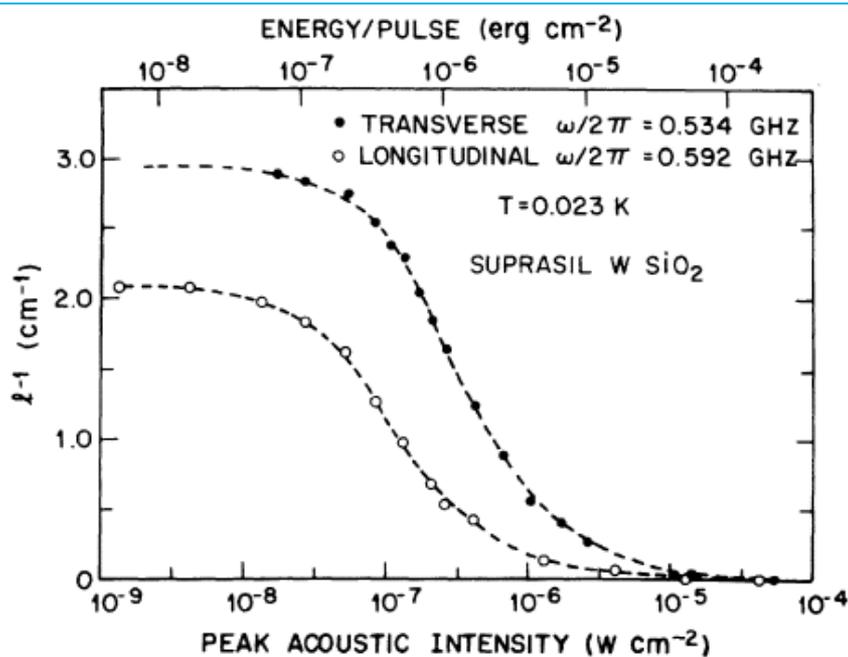
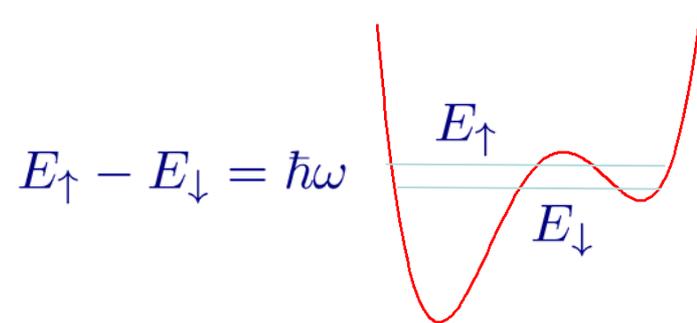


FIG. 1. Reciprocal decay lengths for 0.4- $\mu$ sec phonon pulses in fused silica at 0.023 K as a function of incident intensity.

Resonant TLS



$$l^{-1} \propto n(E_{\downarrow}) - n(E_{\uparrow})$$

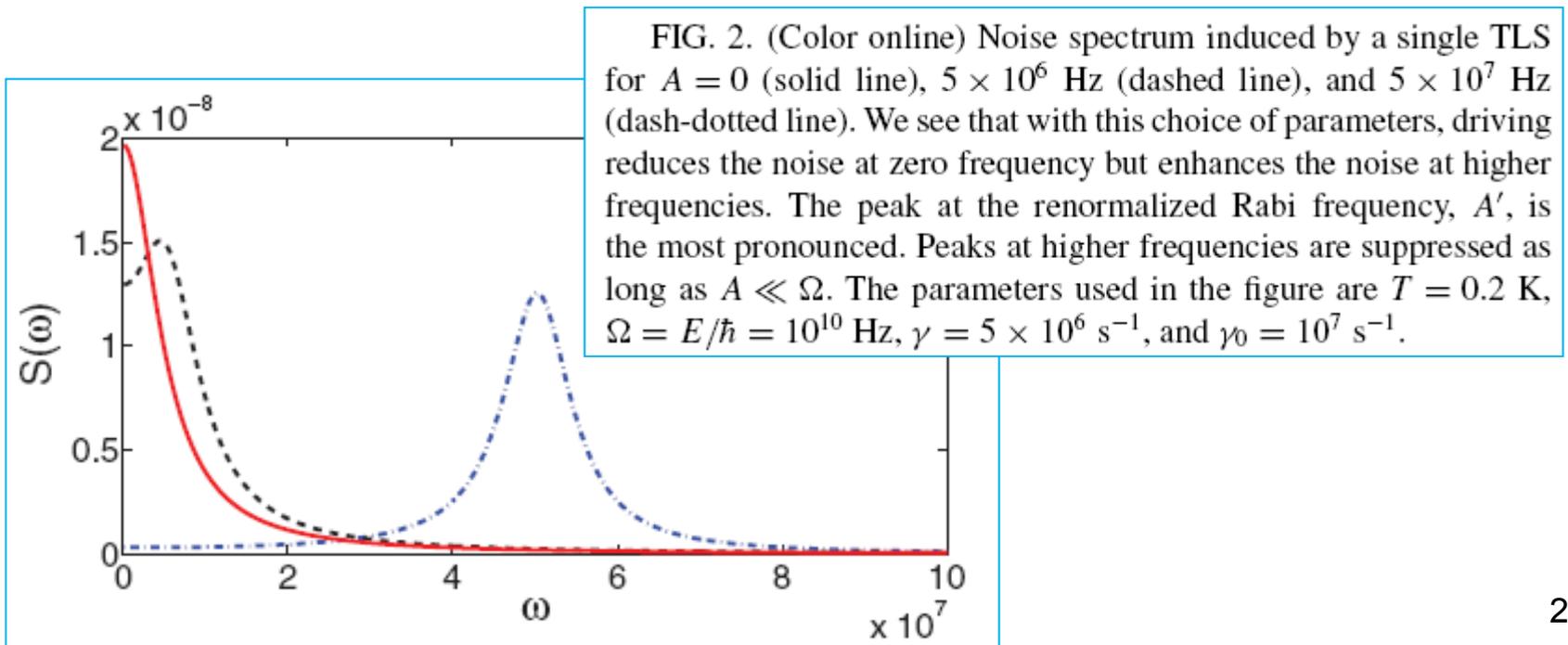
The difference decreases with intensity increase  $\rightarrow$  resonant TLSs become inactive for absorption.

**Hope:** It may be also true for noise

Main conclusion of Constantin et al. - External driving at qubit frequency for typical ensembles of fluctuators will not have significant impact on the low-frequency noise because noise at low frequencies is mostly determined by TLSs with  $E \lesssim kT \ll \hbar\omega$ .

At the same time we [PRB **84**, 245416 (2011)] have shown that driving at relatively low frequencies can modify the noise spectrum decreasing noise at low frequencies.

The net effect of driving on the low- $E$  fluctuators is almost entirely a shift in the frequency spectra from low to high frequencies.



# Back to classical 1/f noise in a non-driven qubit....

According to the model, for the qubit and a fluctuator we have:

$$\mathcal{H} = \frac{1}{2} [E_0 + \mathcal{X}(t)] \sigma_z + \frac{1}{2} F(t) \sigma_x + \frac{1}{2} \sum_i E_i \tau_z^{(i)}$$

where  $E_0 = B + \mathcal{X}_1(0)$ ,  $\mathcal{X}(t) = \mathcal{X}_1(t) - \mathcal{X}_1(0)$ .

Density matrix of the resonantly-manipulated qubit:

$$\hat{\rho} = \begin{pmatrix} n & -if e^{i\Omega t} \\ if^* e^{-i\Omega t} & 1 - n \end{pmatrix}.$$

n and f represent slow dynamics in the rotating frame

## Von Neumann equation:

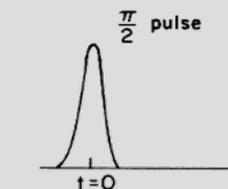
$$\begin{aligned}\frac{\partial n}{\partial t} &= -2\gamma_q(n - n_0) - F \operatorname{Re} f, \\ \frac{\partial f}{\partial t} &= i[E_0 + \mathcal{X}(t) - \Omega]f - \gamma_q f + \frac{F}{2}(2n - 1).\end{aligned}$$

$F$  – Rabi frequency,  $\mathcal{X}(t)$  - random deviation of eigenfrequency

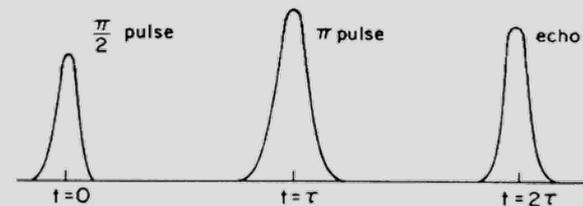
### Stochastic differential equation

The external microwave is applied a set of pulses rotating the effective spin by some angle (manipulation protocol).

Typical protocols -  
profiles  $F(t)$



Free induction

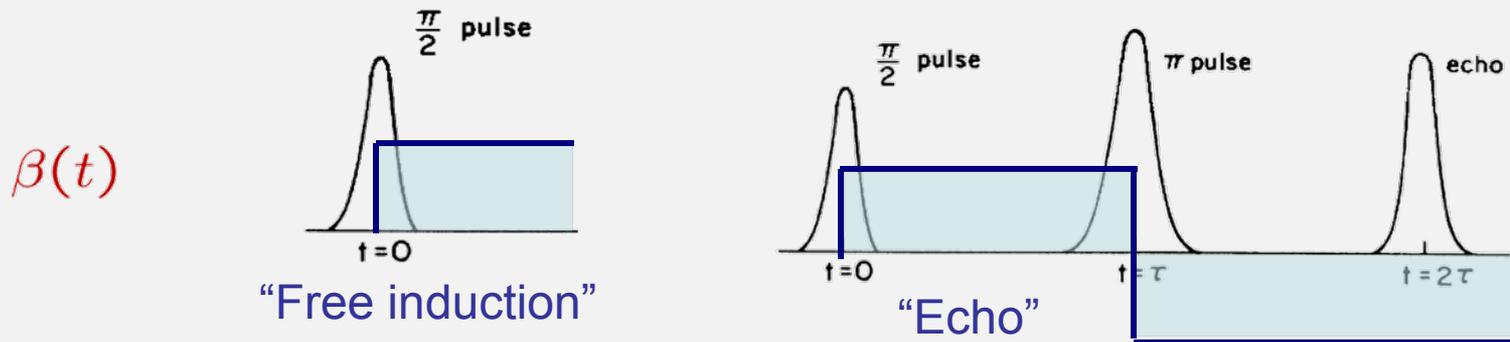


Two-pulse echo

In general, the signal under consideration can be expressed in terms of the phase-memory functional

$$\Psi[\beta(t'), t] = \left\langle \exp \left( i \int_0^t \beta(t') \mathcal{X}(t') dt' \right) \right\rangle_{\xi_i}$$

$\beta(t)$  depends on the manipulation protocol



Similar to spectral diffusion in magnetic systems

R. Klauder and P. W. Anderson, Phys. Rev. **125**, 912 (1962).

and glasses

J. L. Black and B. I. Halperin, Phys. Rev. B **16**, 2829 (1977).

In the latter case fluctuators are structural two-level systems

## Single Fluctuator

Simplification:  $\xi^2(t)=0,1$  is a determined quantity

The phase-memory functional obeys the differential equation

$$\frac{d^2\psi}{d\psi^2} + \left(2\gamma - \frac{d \ln \beta}{dt} - iv\beta\right) - iv\gamma\beta^2 = 0$$

with initial conditions  $\psi(0) = 0$ ,  $\frac{d\psi}{dt} \Big|_{t=0} = \frac{iv}{2}\beta \Big|_{t=-0}$

One can easily find exact solution of a simple manipulation protocol when  $\beta = \pm 1$

Two parameters: switching rate,  $\gamma$ , and coupling strength,  $\nu$

Strong coupling,  $\nu \gg \gamma$ :

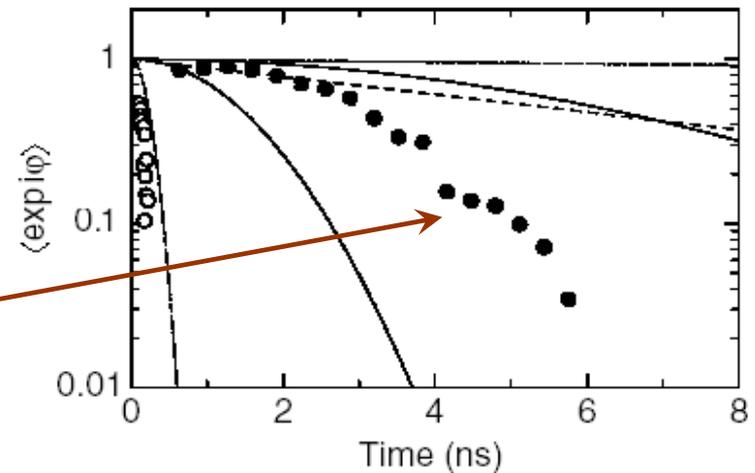
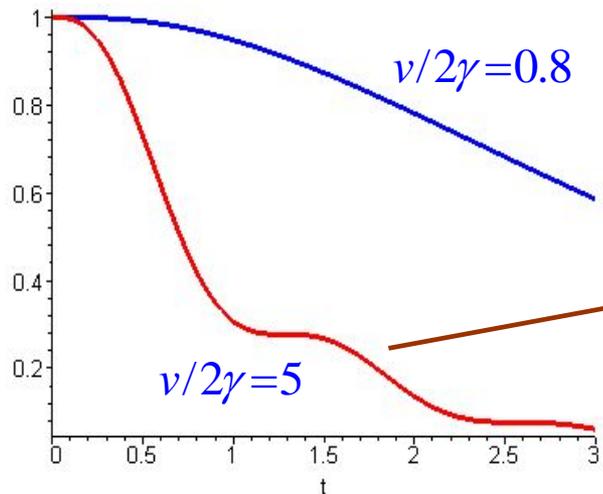
Echo decay

$$\psi(t) = e^{-2\gamma\tau} \left( 1 + \frac{2\gamma}{\nu} \sin \nu\tau \right)$$

Plateaus at  $\nu\tau \approx 2\pi k + 2\gamma/\nu$

Weak coupling,  $\nu \ll \gamma$ :

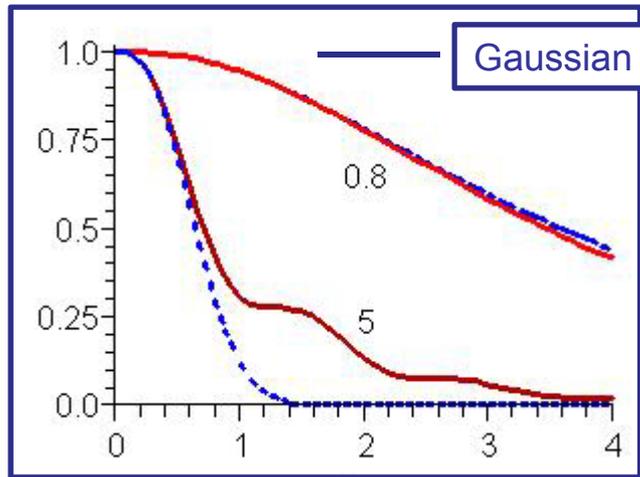
$$\psi(t) = e^{-\nu^2\tau/4\gamma}$$



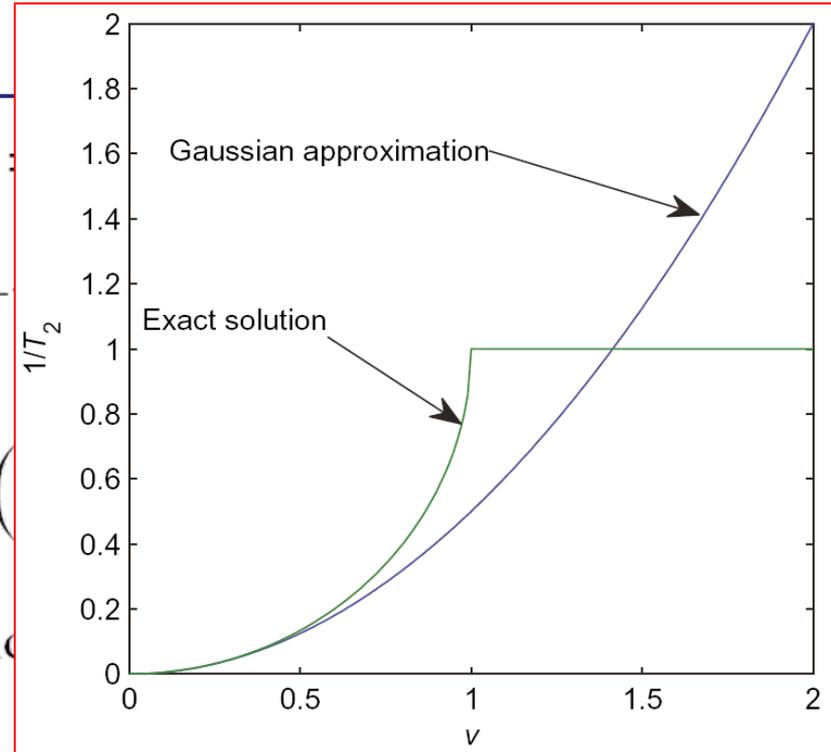
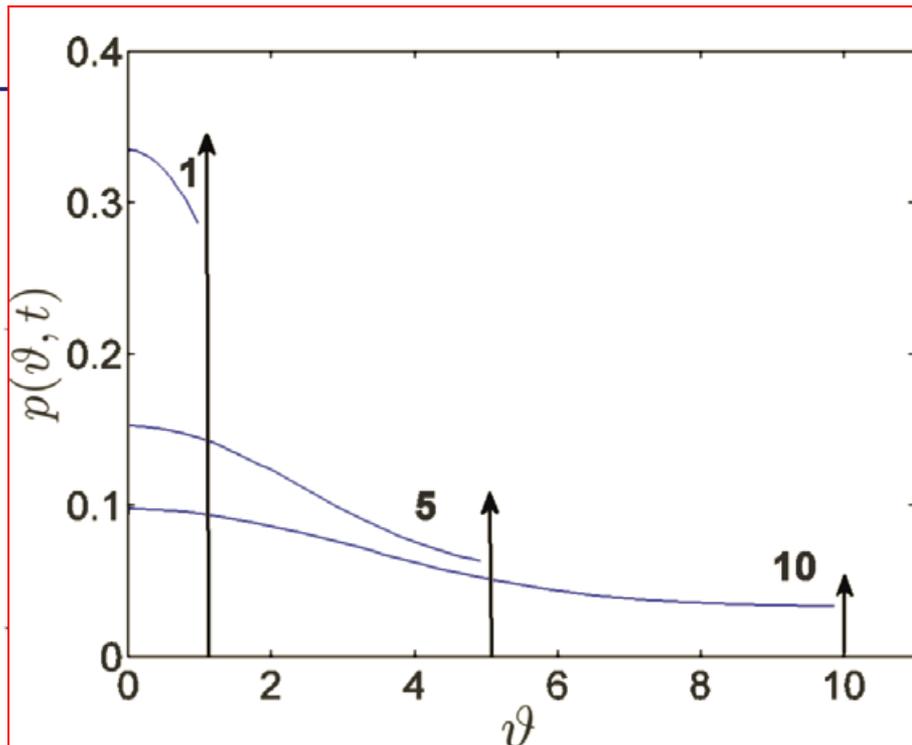
Nakamura et al., PRL 2002

# How good is the Gaussian assumption for a single fluctuator?

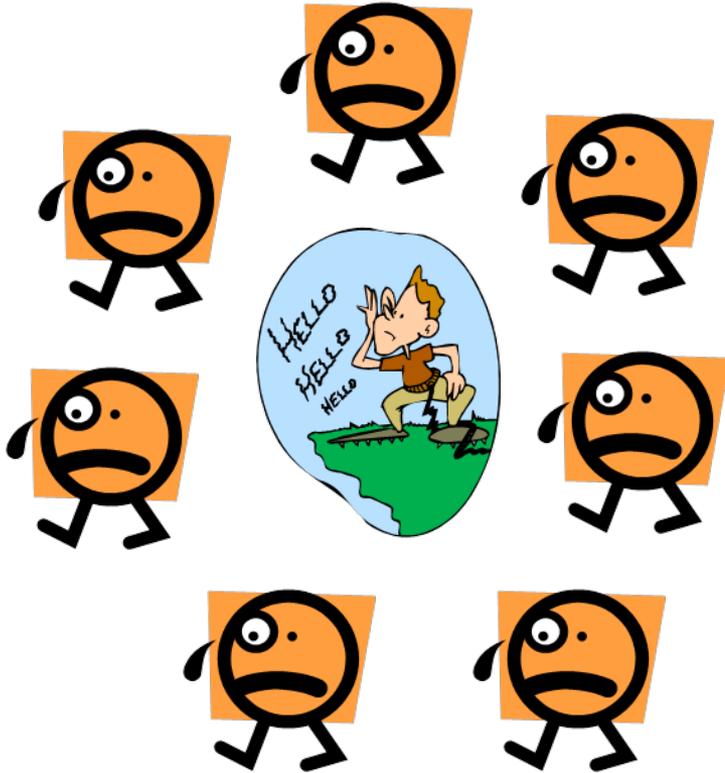
Good only in the case of weak coupling.



**Reason:** The distribution function of the phase shift is essentially non-Gaussian since the phase shift is limited by the quantity  $2\nu\tau$



# Many fluctuators (decoherence by $1/f$ noise)



Many fluctuators with exponentially broad distribution of switching rates produce  $1/f$  noise.

- Can such noise be considered as **Gaussian**?

- What kind of model should describe decoherence by  $1/f$  noise?

- Is the decoherence directly related to the  $1/f$  noise?

Microscopic model leading to **1/f noise** - many uncorrelated fluctuators

$$\Psi(t) = \prod_i \psi^{(i)}(t) = e^{\sum_i \ln \psi^{(i)}(t)}$$

Assuming self-averaging  $\rightarrow e^{N \langle \ln \psi(t) \rangle_F} \equiv e^{-\mathcal{K}(t)}$

For  $N \gg 1$ ,  $\mathcal{K}(t) = N \langle 1 - \psi(t) \rangle_F$

Holtsmark method: S. Chandrasekhar, RMP 15, 1 (1943)

To calculate the average one needs distributions of fluctuators' **decay rates** and **coupling constants**

## Properties of distributions:

- Only the fluctuators with  $E \leq kT$  are important, the rest are frozen in their ground states
- Relaxation rates: since  $\Lambda \propto e^{-\lambda r}$  the distribution of the *logarithm of*  $\Lambda$  should be uniform

$$P(E, \theta) = \frac{P_0}{\sin \theta} \quad \sin \theta \equiv \frac{\Lambda}{E}$$

- Distribution of  $v$  depends both on the interaction range and location of the fluctuators.

In a bulk system, assuming that  $v = g/r^3$  and the fluctuators are *randomly* distributed in *space* we get

$$\mathcal{P}(E, \theta, v) = \frac{\eta}{v^2 \sin \theta}, \quad \eta = \frac{g}{r_T^3}, \quad r_T = (P_0 T)^{-1/3}$$

$\eta$  is the typical coupling to a thermal fluctuator.

General expression:

$$\mathcal{K}(t) = \eta \int \frac{du}{u^2} \int_0^{\pi/2} \frac{d\theta}{\sin \theta} \left\{ 1 - \psi [\beta, t | u \cos \theta, \gamma_0 \sin^2 \theta] \right\}$$

Echo signal:

Switching probability

$$\mathcal{K}(\tau) \sim \begin{cases} \eta\tau(\gamma_0\tau), & \gamma_0\tau \ll 1 \\ \eta\tau, & \gamma_0\tau \gg 1 \end{cases} \begin{array}{l} \text{Markovian} \\ \text{Non-Markovian} \end{array}$$

$$T_2^{-1} \approx \min\{\eta, \sqrt{\eta\gamma_0}\}$$

At  $\tau \gg 1/\gamma_0$  decoherence is due to *optimal* fluctuators with

$$v(r_{\text{opt}}) \approx \gamma_0(T)$$

**Different** from those which mainly contribute to the noise spectrum

# Attempt to compare with experiment

Problem #1: It is the long-time decoherence that is most sensitive to the particular model of the noise. However, at long times the signal usually is weak and obscured by noise.

We have gotten access to extended data from NEC group on echo in flux qubits.

PRL **97**, 167001 (2006)

PHYSICAL REVIEW LETTERS

week ending  
20 OCTOBER 2006

## Decoherence of Flux Qubits due to $1/f$ Flux Noise

F. Yoshihara,<sup>1</sup> K. Harrabi,<sup>2</sup> A. O. Niskanen,<sup>2,3</sup> Y. Nakamura,<sup>1,2,4</sup> and J. S. Tsai<sup>1,2,4</sup>

Problem #2: Despite significant progress (e.g., talk by R. McDermott) the mechanism behind the flux noise is not fully understood.

Therefore we use the simplest model:

$v$  and  $\gamma$  are uncorrelated, distribution of  $v$  is peaked at some value:

$$\mathcal{P}(v, \gamma) = \frac{P_0 T}{\gamma} \delta(v - \bar{v}), \quad \gamma_0 > \gamma > \gamma_{\min}.$$

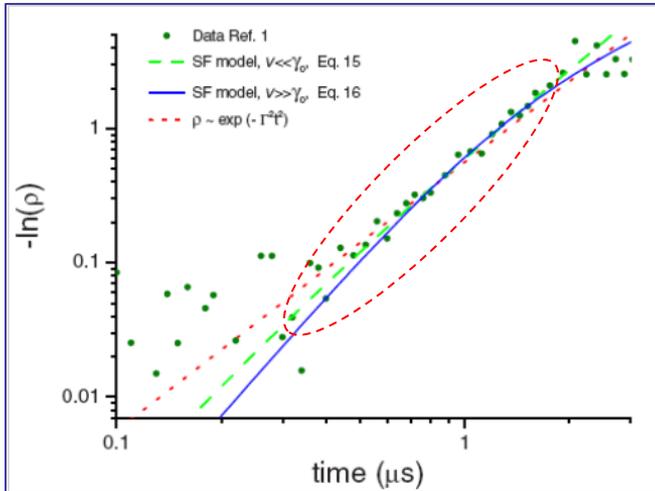
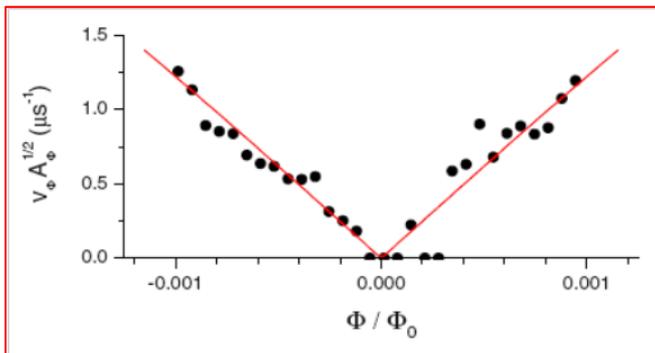


FIG. 1. (Color online) Dephasing component of the echo measurements replotted from Fig. 4(a) of Ref. 1 away from the optimal point. The curves show fits to the SF model, Eqs. (14) and (15), and to the  $\rho = e^{-\Gamma^2 t^2}$  law (Ref. 38). The fitting took into account all data points including those that fall outside the range of the plot (e.g., those with  $\rho > 1$ ).

In the reliable region there is almost no difference between the Gaussian and SF model.



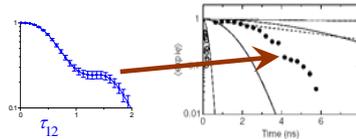
Still, if we use the SF model it is possible to extract the coupling between a typical fluctuator and the qubit.

$$\approx 2 \times 10^{-5} \Phi_0$$

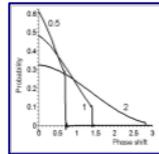
# Summary and Conclusions

We considered extended spin-fluctuator model for

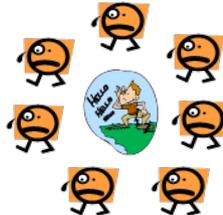
1. Echo & energy relaxation



2. Distribution of single-shot readouts



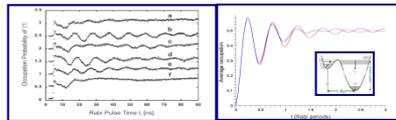
3. Decoherence by many fluctuators



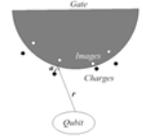
4. Decoherence @ optimal point



5. Fluctuator-mediated Rabi oscillations



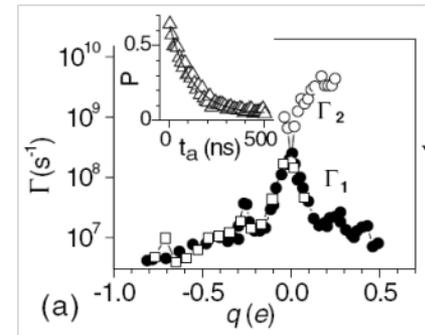
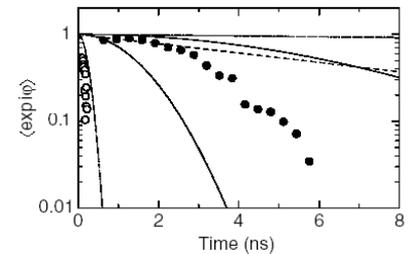
6. Decoherence due to “quenched” fluctuators

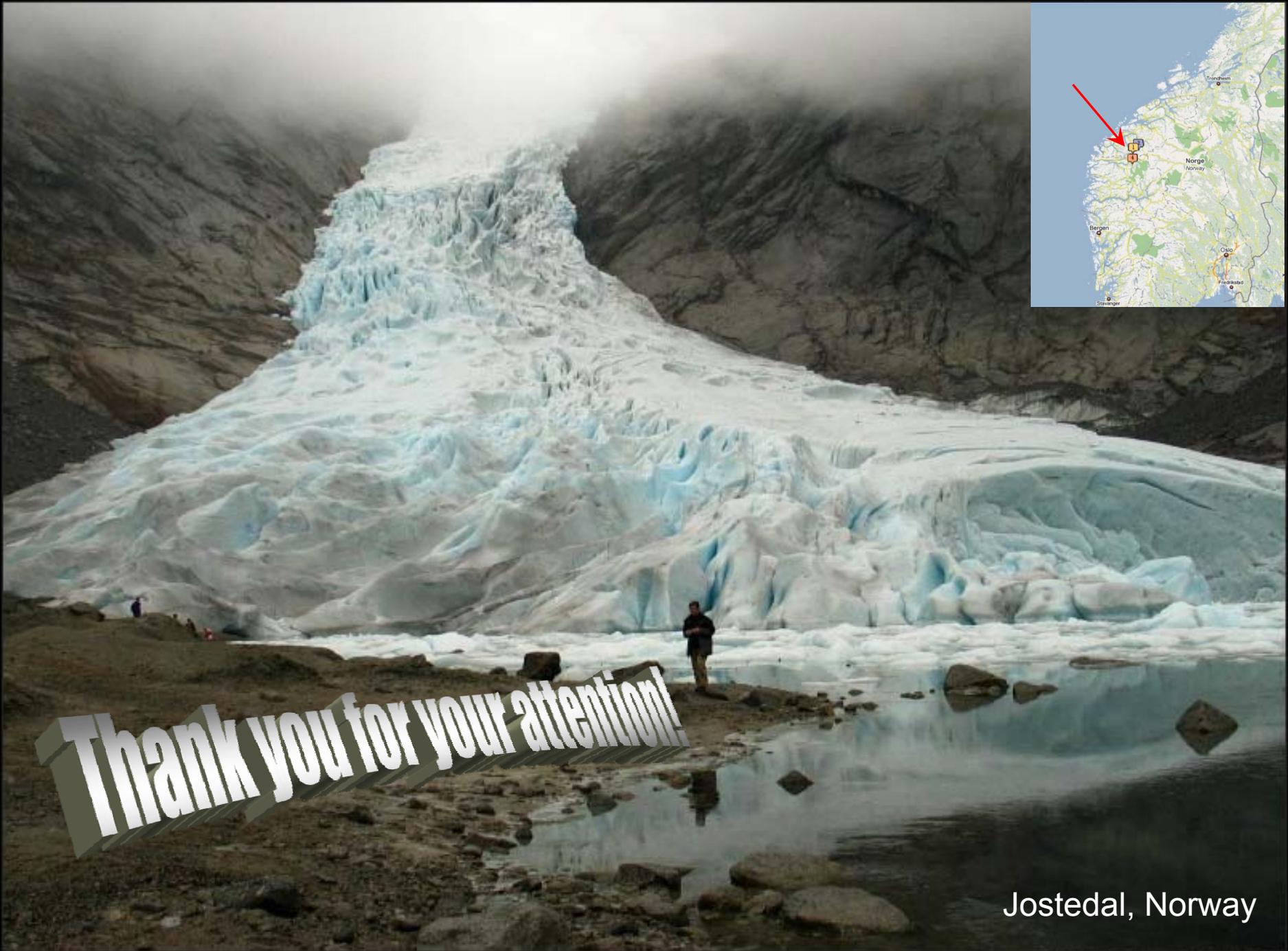


## The model

explains observed features

shows pronounced non-Gaussian behavior – there is NO direct relation between the decoherence and noise spectrum





**Thank you for your attention!**

Jostedal, Norway