Hybridization of wave functions in one-dimensional Anderson localization

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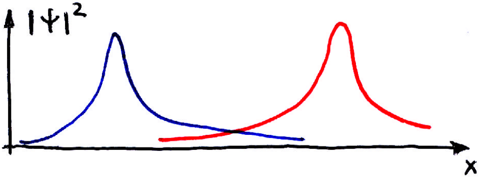
Main result



Mott hybridization argument

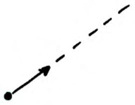
+ log-normal distribution of wave-function tails

= good quantitative method for describing localization

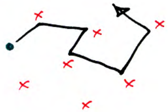
(by comparison with exact 1D and quasi-1D results)

Anderson localization

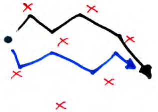
1. Free particle



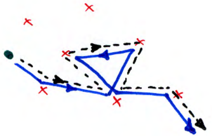
2. Classical diffusion:
$L^2 \propto t$



3. Quantum interference:
$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2\mathrm{Re}\, A_1^* A_2$$



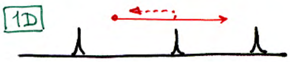
4. Localization corrections:



In 1 and 2 dimensions, interference suppresses the diffusion completely at arbitrary strength of disorder: the particle stays in a finite region of space (localization) [Mott, Twose '61; Berezinsky '73; Abrahams, Anderson, Licciardello, Ramakrishnan '79]

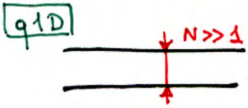
Solvable models in 1D

Particle on a line (strictly 1D):



[Berezinsky technique
+ variations: equations
on the probability distribution
of the scattering phase]

Thick wire (quasi 1D):



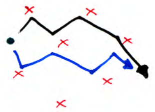
[Efetov's supersymmetric nonlinear sigma model]

Two approaches to describing localization:

- transport properties (transmission coefficients)
- wave-function properties

Quantitative description of localized wave functions

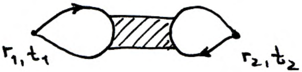
Localization is not visible in the average of a single Green's function:



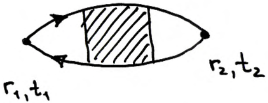
$\langle G(r) \rangle$ decays at the length scale of the mean free path

Averaging two Green's functions (two types of averages):

1. $\langle G(1,1)G(2,2) \rangle$
(correlations of DOS)



2. $\langle G(1,2)G(2,1) \rangle$
(response function)



Correlation functions

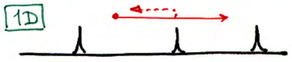
$$R(\omega, x) = \nu^{-2} \Big\langle \sum_{n,m} |\Psi_n(0)|^2 \, |\Psi_m(x)|^2 \, \delta(E_n - E_m - \omega) \, \delta(E - E_n) \Big\rangle$$

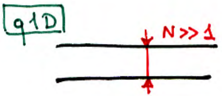
$$S(\omega, x) = \nu^{-2} \Big\langle \sum_{n,m} \Psi_n^*(0) \Psi_n(x) \Psi_m^*(x) \Psi_m(0) \times \delta(E_n - E_m - \omega) \, \delta(E - E_n) \Big\rangle$$

Averaging is over disorder realizations

1D models

- Strictly 1D (S1D)
- Quasi-1D unitary (Q1D-U)
- Quasi-1D orthogonal (Q1D-O)





Assumptions:

- Gaussian white-noise disorder
- $kl \gg 1$ (l – mean free path)

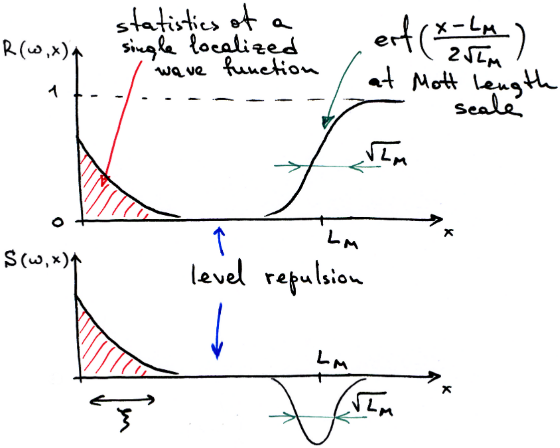
Then the localization length is

$\xi \sim l$ in S1D

$\xi \sim Nl$ in Q1D ($N \gg 1$ – number of channels)

Energy scale: Δ_ξ — level spacing AND Thouless energy at $x \sim \xi$.

Correlations in the localized regime ($\omega \ll \Delta_\xi$)



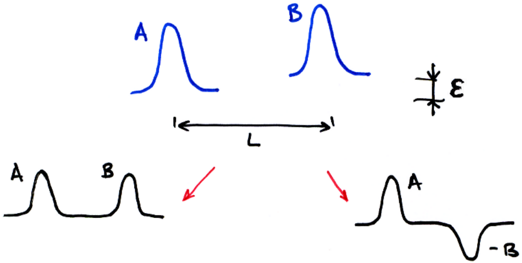
qualitatively explained by Mott hybridization argument

$L_M \sim \log(\Delta_\xi/\omega)$
– Mott length scale

[Gor'kov, Dorokhov, Prigara '83: S1D]
[DI, Ostrovsky, Skvortsov '09: $R(\omega, x)$ in Q1D-U]

Mott argument (wave function hybridization)

At $\omega \ll \Delta_\xi$, main contribution to correlations comes from pairs of hybridized states:



1. At short distances ($x \lesssim \xi$), the two eigenfunctions have the same profile (single localized wave function)

2. Hybridization is important as long as the splitting

$$\Delta_\xi \exp(-L/2\xi) > \omega \quad \Leftrightarrow \quad L < L_M = 2\xi \ln(\Delta_\xi/\omega)$$

Mott argument: quantitative approach

1. Averaging over all possible positions of the two hybridizing states and over the relative energy difference ε.

2. Diagonalize $\begin{pmatrix} \varepsilon/2 & J^* \\ J & -\varepsilon/2 \end{pmatrix} \quad \Rightarrow \quad \omega = \sqrt{\varepsilon^2 + 4|J|^2}$

3. Simplest guess (to be corrected): $J = \Delta_\xi e^{-x/2\xi}$
[Sivan, Imry '87]

– Explains qualitatively, but not quantitatively, e.g. fails to explain

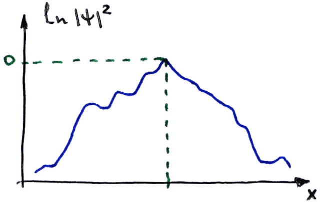
- width $\sqrt{L_M \xi}$ of the feature at $x \sim L_M$
- $|\Psi_n(0)|^2 |\Psi_n(x)|^2 \propto e^{-x/4\xi}$ instead of $e^{-x/\xi}$

Can be repaired, if the log-normal distribution of tails is taken into account

Log-normal tails (phenomenological rules 1)

1. Wave-function decomposition [Kolokolov '95, Mirlin '00]:
$\Psi(x) = \tilde{\Psi}(x) \cdot \varphi(x)$ (envelope · short-range oscillations)

2. Envelope $\chi(x) = \ln|\tilde{\Psi}(x)|^2$ obeys diffusion + drift equation (at large distances from maximum $r = |x|/\xi \gg 1$):



$$\frac{\partial P}{\partial r} = \frac{\partial^2 P}{\partial \chi^2} + \frac{\partial P}{\partial \chi}$$

$$P(\chi = 0) = 0$$

$$\Rightarrow \quad P(\chi, r \to \infty) = f\left(\frac{\chi}{r}\right) \frac{1}{2\sqrt{\pi r}} \exp\left[-\frac{(\chi + r)^2}{4r}\right]$$

Log-normal tails (phenomenological rules 2)

3. Ansatz for hybridization matrix element
(by analogy with the two-well problem):

$$|J| = \Phi \, \tilde{\Psi}_A(x) \, \tilde{\Psi}_B(x)$$

Φ has its own distribution and distinguishes between S1D and Q1D and between symmetry classes in Q1D (by analogy with random matrices):

$$dP(\Phi) = \delta(\Phi - \Phi_0) \, d\Phi \quad \text{with} \quad \Phi_0 \sim 1 \quad \text{(S1D)}$$
$$dP(\Phi) \propto \Phi \, d\Phi \,, \quad \Phi \to 0 \quad \text{(Q1D-U)}$$
$$dP(\Phi) \propto d\Phi \,, \quad \Phi \to 0 \quad \text{(Q1D-O)}$$

These rules reproduce remarkably well exact results in 1D (including the first subleading correction)

Comparison with exact results and new conjectures

DOS correlation function $R(\omega, x) = \nu^{-2}\langle \rho_E(0)\rho_{E+\omega}(x)\rangle$

Model	$R(\omega{=}0, x{\gg}1)$	$\delta R(\omega, x{\gg}1)$	at Mott length
1D	✓	$\omega^2(L_M - 3x)e^{2x}$	✓
Q1D-U	✓ $x^{-3/2}e^{-x/4}$	✓ $\omega^2(L_M - 3x)^2 e^{2x}$	✓ $\frac{1}{2}\left(1 + \operatorname{erf}\frac{x-L_M}{2\sqrt{x}}\right)$
Q1D-O		$\omega e^{-x/2}$	

Dynamical response function $S(\omega, x) = \nu^{-2}\langle G^R_E(0, x)G^A_{E+\omega}(x, 0)\rangle$

Model	$S(\omega{=}0, x{\gg}1)$	$\delta S(\omega, x{\gg}1)$	at Mott length
1D	✓	$-\omega^2(L_M - 3x)e^{2x}$	✓
Q1D-U	$x^{-3/2}e^{-x/4}$	$-\omega^2(L_M - 3x)^2 e^{2x}$	$-\frac{\exp\left[-\frac{(x-L_M)^2}{4x}\right]}{2\sqrt{\pi x}}$
Q1D-O		$-\omega e^{-x/2}$	

(✓ mark available exact results)

Summary and possible applications

- Hybridization of log-normally distributed tails:
  an easy approximation to study localized states,
  much simpler than exact methods
- New results (conjectures) for quantum wires
  in the orthogonal symmetry class

Possible extensions:

- away from one-parameter scaling (strong disorder)
- to higher dimensions
- to wires with a finite number of channels (crossover from $N = 1$ to $N = \infty$)
- to contacts between Anderson insulators and superconductors