Non-adiabatic Josephson dynamics in junctions with in-gap quasiparticles

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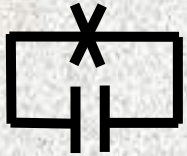
- Motivation
- Adiabatic vs non-adiabatic dynamics
- D-wave junctions: generic case
- Non-adiabatic equations
- Linear case, role of MGS
- Application to TA-MQT transition
- Non-linear case

Adiabatic (conventional) Josephson dynamics

Josephson Junction = nonlinear oscillator



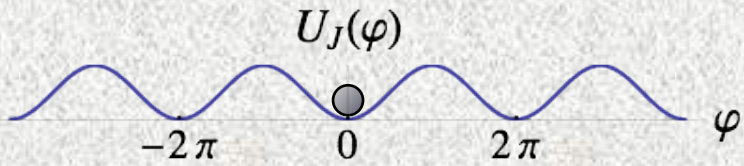
Displacement current

Quasi-static Josephson current

$$\frac{C}{2e}\ddot{\varphi} + I_J(\varphi) = I_{\text{ext}}(\varphi)$$

Particle in a potential

$$I_J(\varphi) = 2e\frac{\partial U_J}{\partial \varphi}$$



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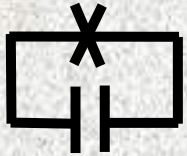


Displacement current

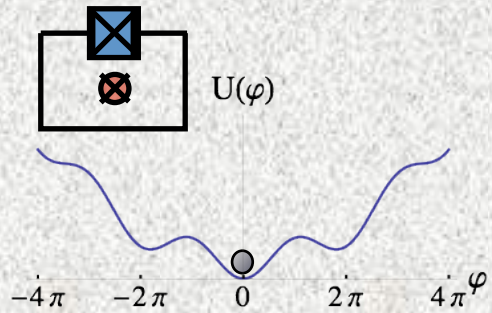
Quasi-static Josephson current

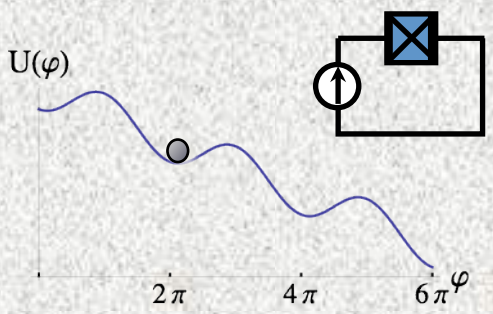
$$\frac{C}{2e}\ddot{\varphi} + I_J(\varphi) = I_{\text{ext}}(\varphi)$$

Particle in a potential

$$I_J(\varphi) = 2e\frac{\partial U_J}{\partial \varphi}$$

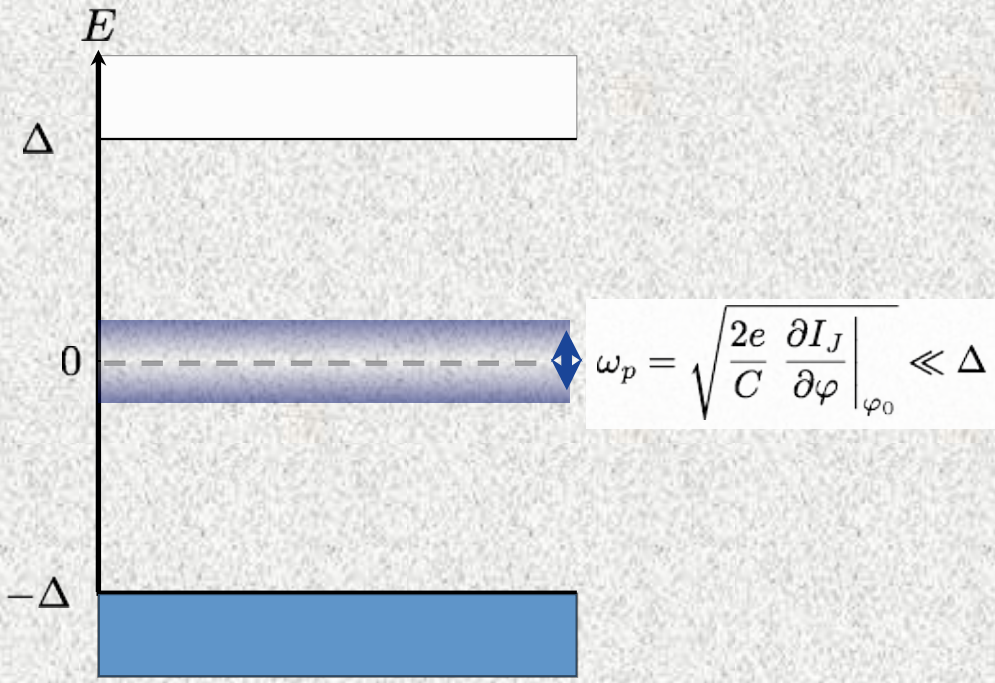
$$I_{\text{ext}}(\varphi) = -2e\frac{\partial U_{\text{ext}}}{\partial \varphi}$$





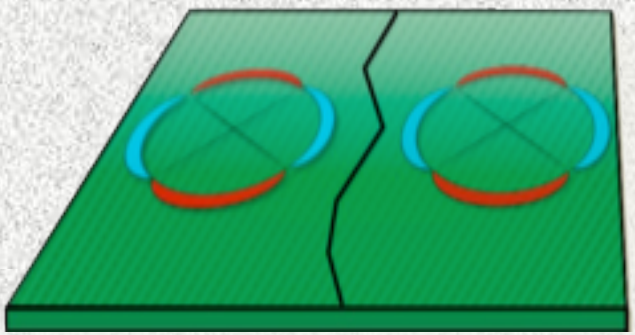
Adiabatic (conventional) Josephson dynamics

Microscopic degrees of freedom (quasiparticles)
are "frozen out"

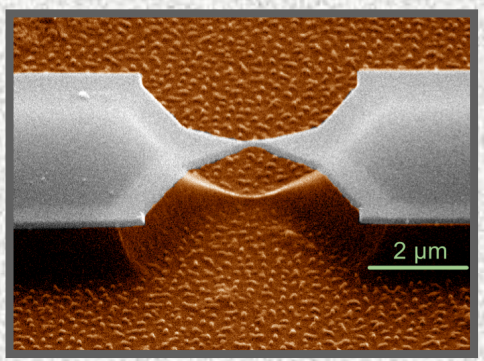


Josephson junctions with in-gap quasiparticles

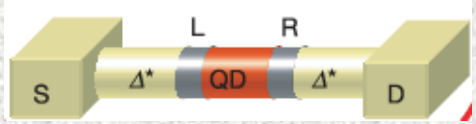
- Josephson junctions composed of superconductors with unconventional order parameters
- Constriction type Josephson junctions with high transparency channels
- Josephson junctions with quantum dots (resonant states)
- Spurious low energy two-level systems located in the junction



D-Wave



QPC

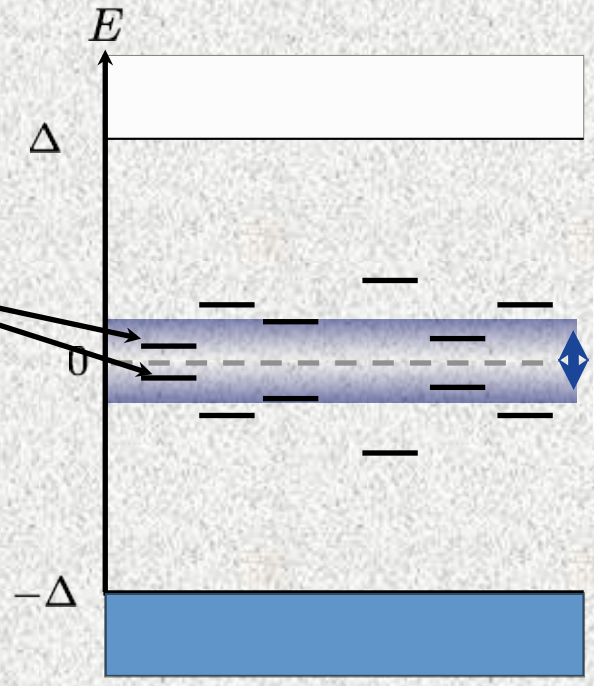


NW / CNT

Nonadiabatic Josephson dynamics

Microscopic degrees of freedom (quasiparticles) become involved in the dynamics.

Low energy Andreev Levels



$$\omega_p = \sqrt{\frac{2e}{C}\left.\frac{\partial I_J}{\partial \varphi}\right|_{\varphi_0}} \ll \Delta$$

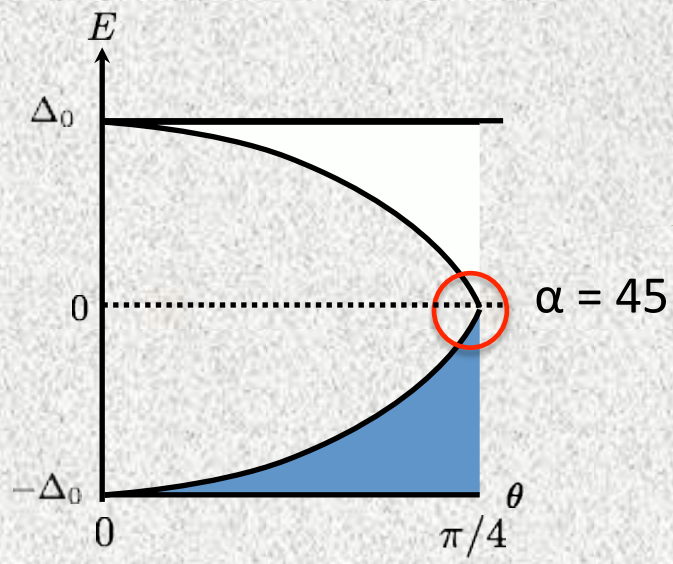
Quasi-static Josephson current

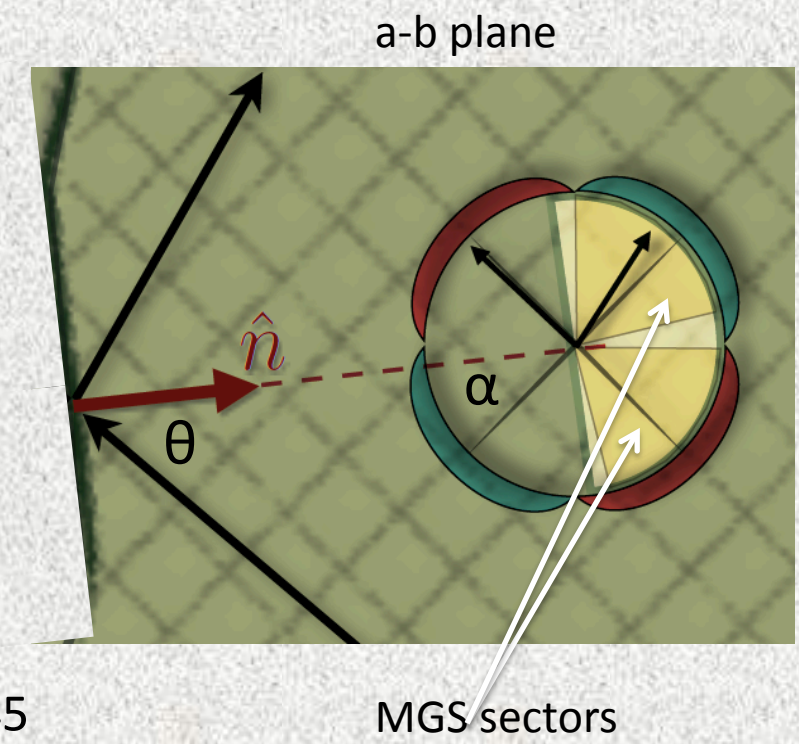
$$\frac{C}{2e}\ddot{\varphi} + \cancel{I_J(\varphi)} = I_{\text{ext}}(\varphi)$$

D-wave superconductors

$$\Delta(\theta) = \Delta_0 \cos 2(\alpha - \theta)$$

- Nodal itinerant states

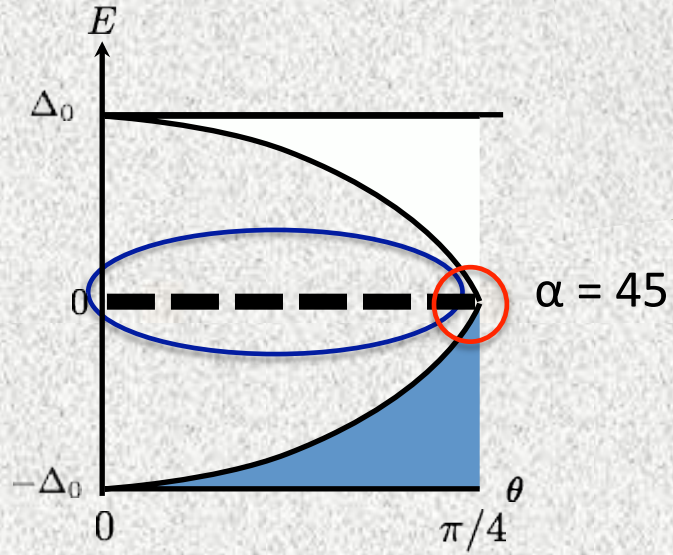


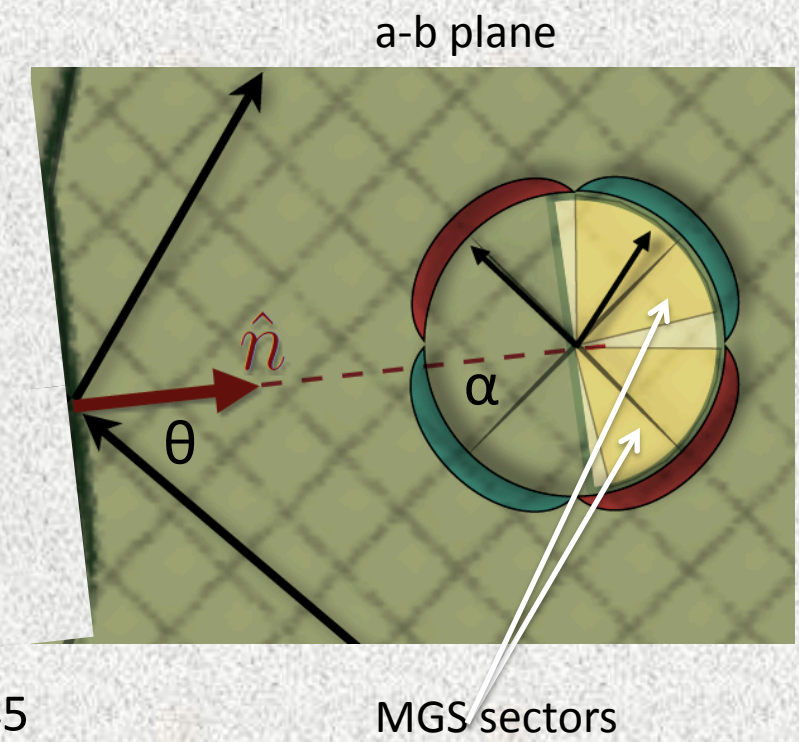


D-wave superconductors

$$\Delta(\theta) = \Delta_0 \cos 2(\alpha - \theta)$$

- Nodal itinerant states
- Zero energy edge states (midgap states)

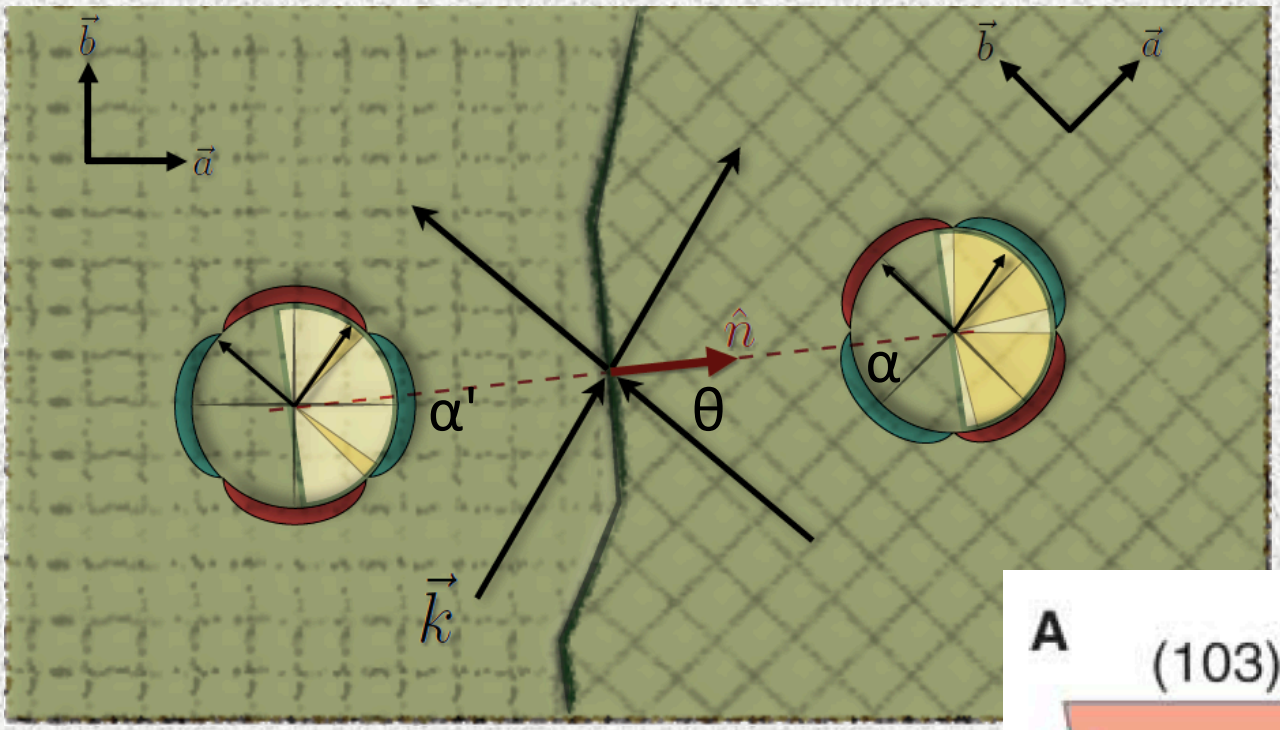




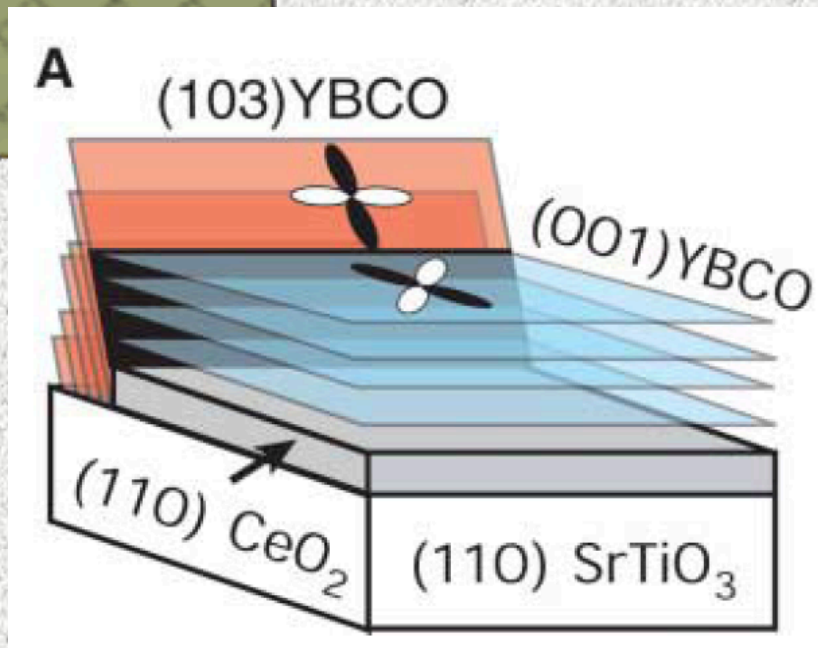
MGS have large phase volume

C. -R. Hu. *Phys. Rev. Lett.* 72, 1526 (1994)

D-wave Josephson junctions



Bauch PRL **94**, 087003 (2005)

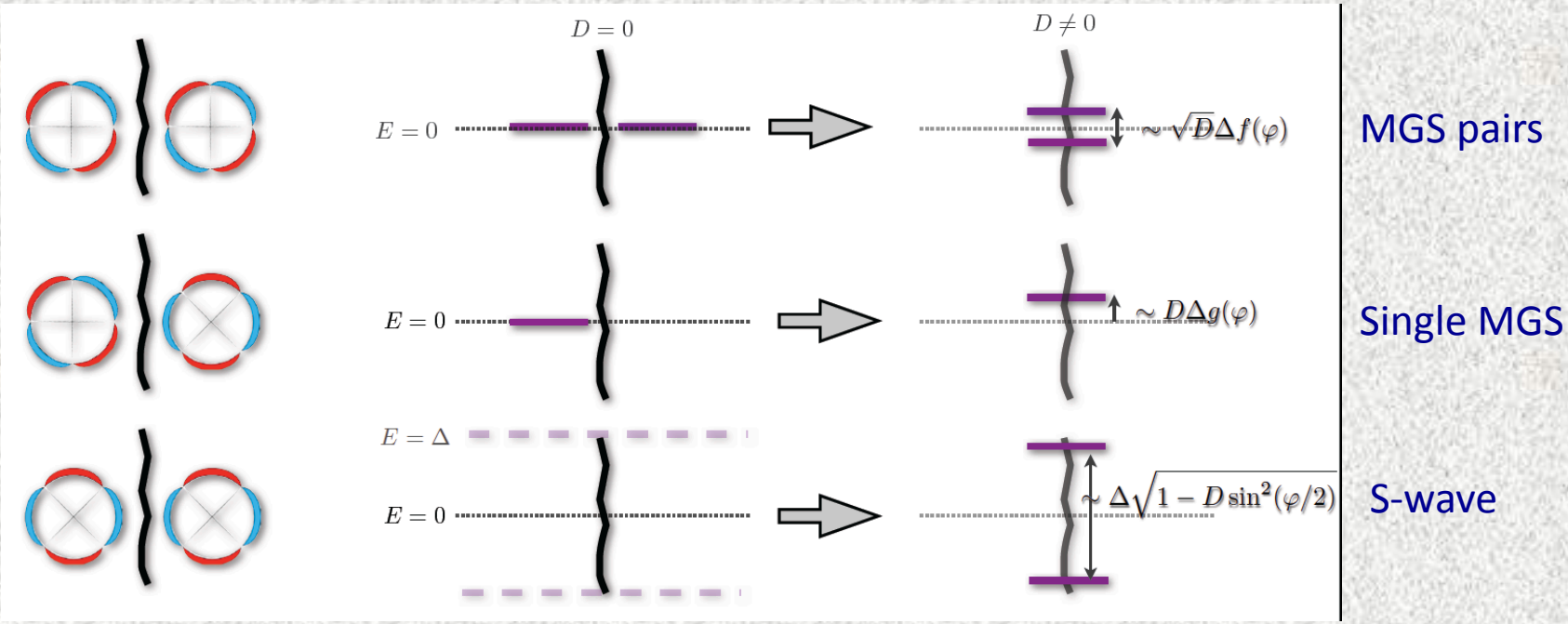


Artificial Grain Boundary Junction

High quality AGBJ:

- specular reflection at relatively long boundary segments (facets)
- however, many facets in the junction

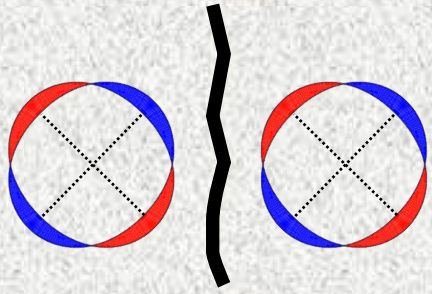
D-wave Josephson junctions



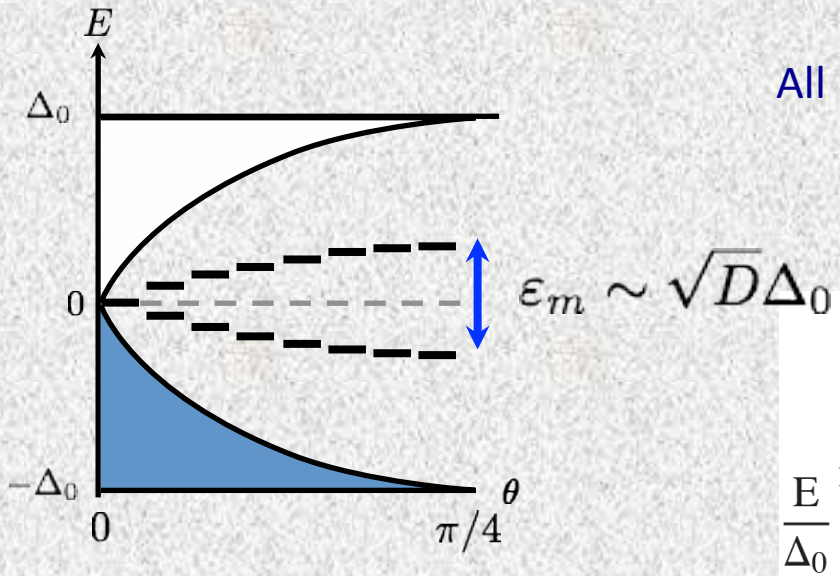
In practice all these possibilities are realized in AGBJ due to faceting

D-wave Josephson junctions

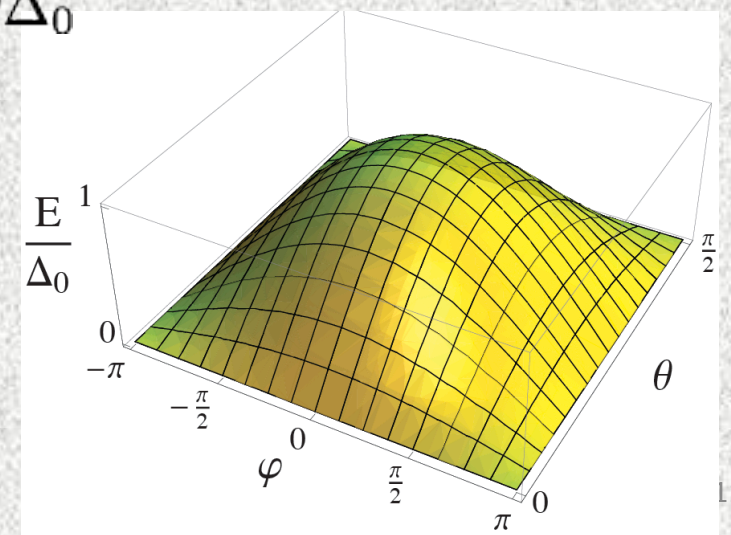
$$\Delta(\theta) = \Delta_0 \cos 2(\alpha - \theta)$$



- D-wave junction - band of midgap states



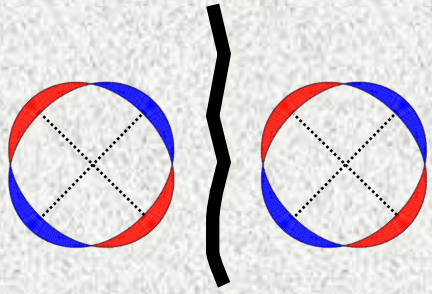
All MGS carry Josephson current



T. Löfwander et al., Supercond. Sci. Techn **14**, R53 (2001)

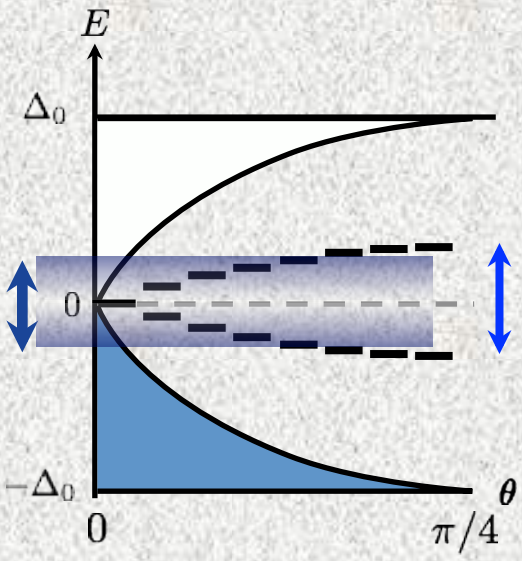
D-wave Josephson junctions

$$\Delta(\theta) = \Delta_0 \cos 2(\alpha - \theta)$$



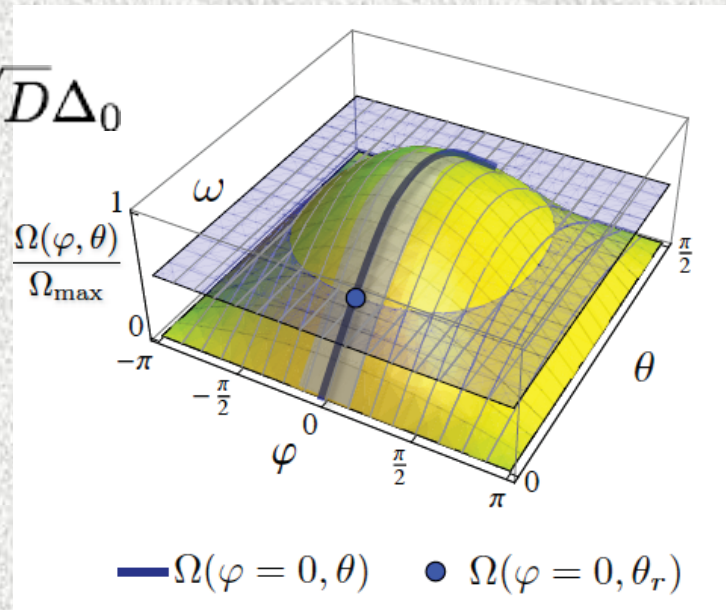
- Resonant interaction of midgap states with Josephson dynamics

Resonance
$\omega_p < \varepsilon_m$

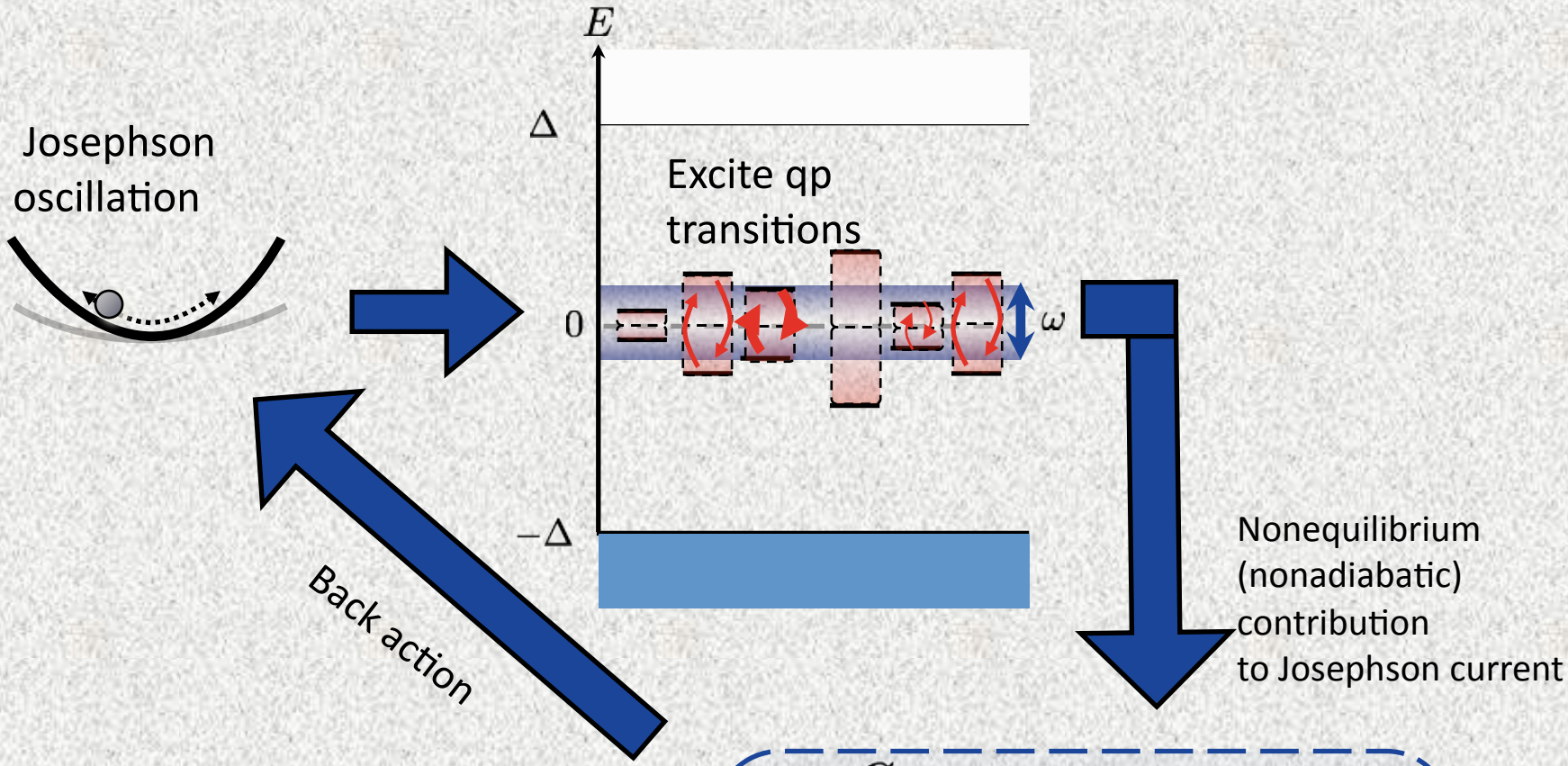


$\varepsilon_m \sim \sqrt{D}\Delta_0$

ω_p



Resonant interaction with in-gap states



$$\frac{C}{2e}\ddot{\varphi} + \mathrm{Tr}\left(\hat{I}\hat{\rho}\right) = I_{\mathrm{ext}}$$

Generalization of conventional Josephson dynamics - including in-gap quasiparticle states

$$\frac{C}{2e}\ddot{\varphi} + I_J(\varphi) = I_{\text{ext}}(\varphi)$$

$$\frac{C}{2e}\ddot{\varphi} + \text{Tr}\left(\hat{I}\hat{\rho}\right) = I_{\text{ext}}$$

$$i\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$$

Josephson dynamics is coupled to dynamics of low energy quasiparticles, represented by **Liouville equation** for quasiparticle **density matrix**

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Josephson dynamics is coupled to dynamics of low energy quasiparticles, represented by **Liouville equation** for quasiparticle **density matrix**

$$\hat{I} = 2e\left(\frac{\partial \hat{E}}{\partial \varphi} + i[\hat{E}, \hat{A}]\right)$$

Josephson current operator

$$\hat{H} = \hat{E} - \dot{\varphi}\hat{\mathcal{A}}$$

QP energy

Interlevel matrix element

J. Michelsen, VS, Low Temp. Phys. **36**, 925 (2010)
PRL, **105,** 127001 (2010)

General derivation scheme

Microscopic contact description

$$Z = \mathrm{Tr}[\hat{\rho}(t)] = \int \mathcal{D}\varphi \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS[\varphi,\bar{\psi},\psi]}$$

$$S[\varphi,\bar{\psi},\psi] = \int_{\mathcal{C}} dt \left(\frac{C}{8e^2}\dot{\varphi}^2 - U_e(\varphi) + (\bar{\psi}, \mathcal{G}^{-1}\psi) \right)$$

$$\mathcal{G}^{-1} = i\partial_t - \mathcal{H}(\varphi) - \frac{\dot{\chi}}{2}\sigma_z$$

$$\mathcal{H}(\varphi) = \left(\frac{\mathbf{p}^2}{2m} - E_F + V(\mathbf{r}) \right)\sigma_z + \Delta e^{i\sigma_z \chi}\sigma_x$$

time dependent SC phase

Instantaneous basis

$$\mathcal{H}\phi_i = E_i\phi_i$$

$$\psi(\mathbf{r},t) = \sum_i \phi_i(\mathbf{r};\varphi)a_i(t)$$

$$S[\varphi,\{\bar{a}_i\},\{a_i\}] = \int_C dt\left(\frac{C}{8e^2}\dot{\varphi}^2 - U_e(\varphi) + \sum_{ij}\bar{a}_i G_{ij}^{-1} a_j\right)$$

$$G_{ij}^{-1} = (i\partial_t - H_{ij})$$

$$H_{ij}(\varphi,\dot{\varphi}) = E_i(\varphi)\delta_{ij} - \dot{\varphi}\mathcal{A}_{ij}$$

instantaneous QP energy

interlevel matrix element

$$\mathcal{A}_{ij} = (\phi_i, i\partial_\varphi\phi_j) - \frac{1}{4}(\phi_i, \mathrm{sign}(x)\sigma_z\phi_j)$$

Dynamics of QP

Density matrix $\hat{\rho}(t) = (1/2i)\sum_a \hat{G}^{aa}(t,t)$

$$i\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}], \qquad \hat{H} = \hat{E} - \dot{\varphi}\,\hat{\mathcal{A}}$$

Dynamics of phase

Least action principle $\left.\frac{\delta S_{\text{eff}}[\varphi,\chi]}{\delta\chi}\right|_{\chi=0} = 0$ $\quad \varphi^a = \varphi + a\chi/2$

$$\frac{C}{2e}\ddot{\varphi} + \mathrm{Tr}(\hat{I}_J\hat{\rho}) = I_e$$

$$\hat{I}_J = 2e(\partial_\varphi \hat{E} + i[\hat{E},\hat{\mathcal{A}}])$$

Josephson junctions with in-gap quasiparticles

$$\frac{C}{2e}\ddot{\varphi} + \mathrm{Tr}\left(\hat{I}\hat{\rho}\right) = I_{\mathrm{ext}}$$

$$i\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$$

$$\hat{I} = 2e\left(\frac{\partial \hat{E}}{\partial \varphi} + i[\hat{E}, \hat{A}]\right)$$

$$\hat{H} = \hat{E} - \dot{\varphi}\hat{\mathcal{A}}$$

Full quantum Hamiltonian (phase + QP)

$$\mathcal{H}_q = \frac{(2e)^2}{2C}\left(\hat{n} + \sum_{ij} \mathcal{A}_{ij}(\varphi)\hat{a}_i^\dagger \hat{a}_j\right)^2 + U_e(\varphi) + \sum_i E_i(\varphi)\hat{a}_i^\dagger \hat{a}_i$$

J. Michelsen, VS, Low Temp. Phys. **36**, 925 (2010)
PRL, **105,** 127001 (2010)

Linear quasiparticle response (dissipative bath)

Consider small deviation from an equilibrium configuration $\varphi_0, \hat{\rho}_0$

Dispersion equation: $$\left(-\omega^2 + \omega_p^2 + \omega\gamma_0(\omega)\right)\varphi_\omega = 0$$

$$\omega_p^2 = \frac{2e}{C}\frac{\partial I_J^{\mathrm{ad}}(\varphi_0)}{\partial\varphi}$$

$$\gamma_0(\omega) = \frac{(2e)^2}{C}\sum_{ij}\frac{\varepsilon_{ij}|\mathcal{A}_{ij}|^2\left(\rho_i^0 - \rho_j^0\right)}{\varepsilon_{ij} - (\omega + i0)}$$

$$\varepsilon_{ij} = E_i - E_j$$

- **Linear response is relevant when bath remains close to equilibrium**
  - itinerant states in the nodal regions of high-Tc superconductors
  - broadened AL in disordered junctions

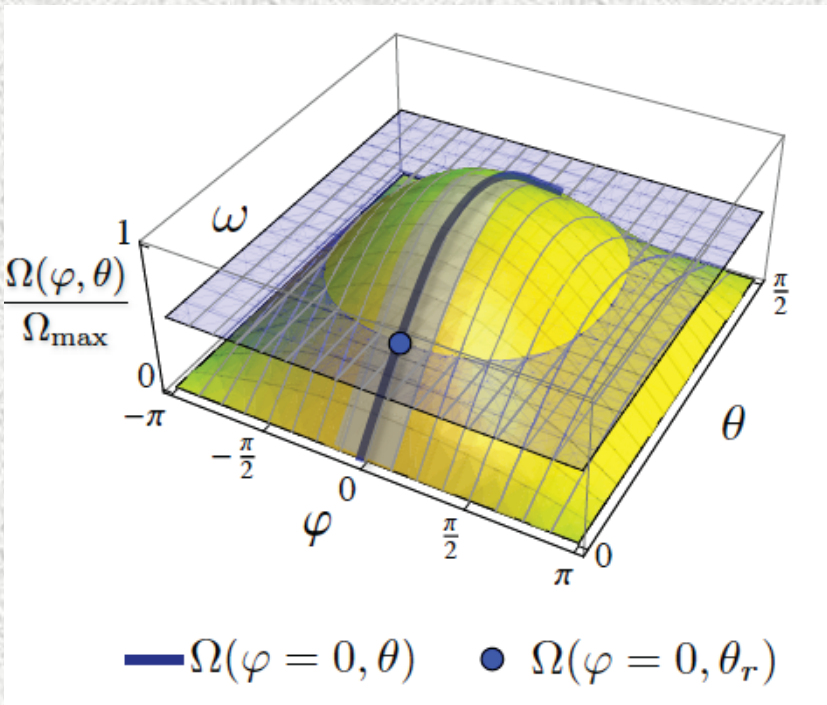
- Phase variation only induces transitions among the states of the same conducting mode

MGS contribution

Two quantities are to be calculated: MGS energy and transition matrix element

Dispersion equation

$$\prod_{\sigma=\pm} \sin\left[\gamma_L^{\sigma}(E) - \gamma_R^{\sigma}(E)\right] = R \prod_{\sigma=\pm} \sin\left[\gamma_L^{\sigma}(E) - \gamma_R^{-\sigma}(E)\right]$$



$\gamma(E,D,\alpha,\theta)$ = known function of junction parameters,
D = 1-R interface transparency
Along qp trajectory

$$\varepsilon_m \propto \sqrt{D}\Delta_0$$

T. Löfwander et al., Supercond. Sci. Techn **14**, R53 (2001)

Matrix element

$$\mathcal{A} \equiv i\mathcal{A}_{12} = \frac{1}{2e}\frac{I_{12}}{(E_1 - E_2)}$$

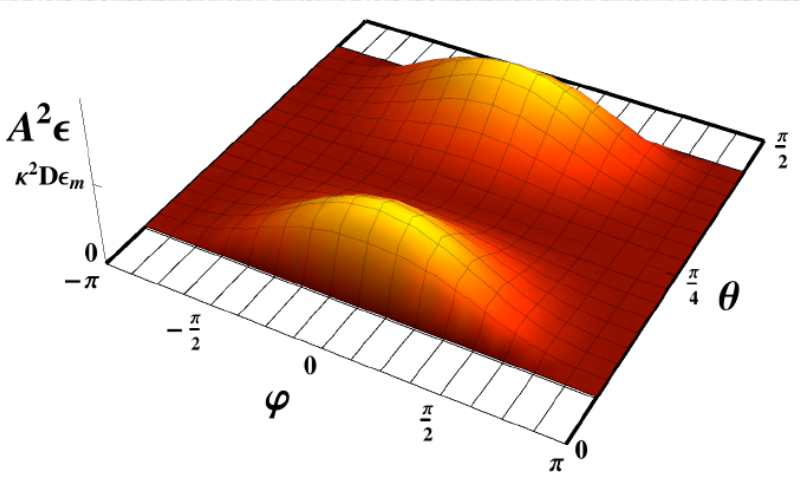
$$I_{12} = e\sum_{\sigma=\pm}(\mathbf{v}_F^\sigma \cdot \mathbf{n})[B_1^\sigma]^* B_2^\sigma \cos\left[\eta_R^\sigma(E_1) - \eta_R^\sigma(E_2)\right]$$

known functions of E

Selection rule : $\mathcal{A}_{12}(\alpha,\alpha) = 0$

"Antisymmetric" misorientations
(π/4 + κ) / (π/4 – κ)

$$\mathcal{A}_{12} \approx 2\kappa\sqrt{D}\cot(2\theta)$$



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Linear dissipation

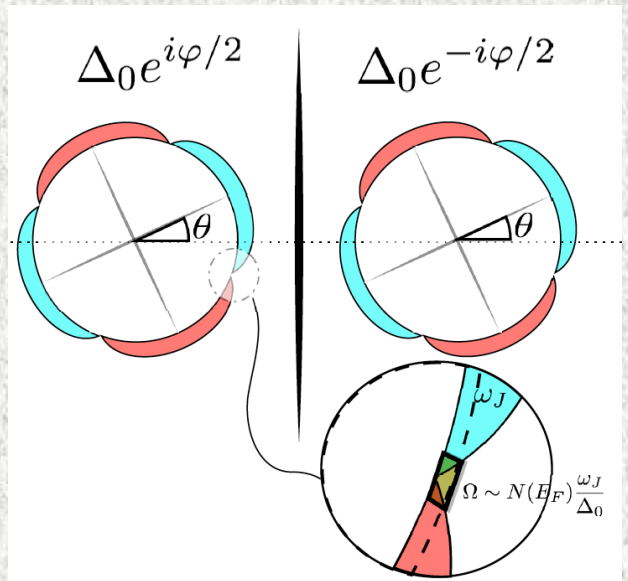
Dispersion equation: $\left(-\omega^2 + \omega_p^2 + \omega\gamma_0(\omega)\right)\varphi_\omega = 0$

Contribution of MGS $\gamma_0''(\omega) \sim \frac{\kappa^2}{R_n C}\left(\frac{\omega}{\varepsilon_m}\right)\tanh\frac{\omega}{4T}$ $R_n = \pi/e^2 S\langle D\rangle$

saturation at
ω >> T

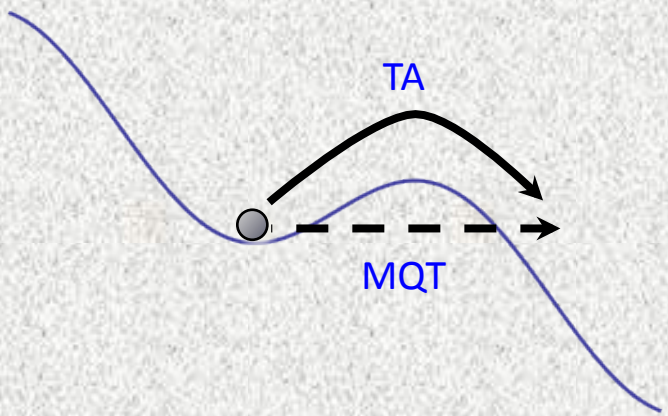
Contribution of nodal QP $\gamma''_{nod} \sim \gamma''_{nod-MGS} \sim \frac{1}{R_n C}\left(\frac{\omega}{\Delta_0}\right)$

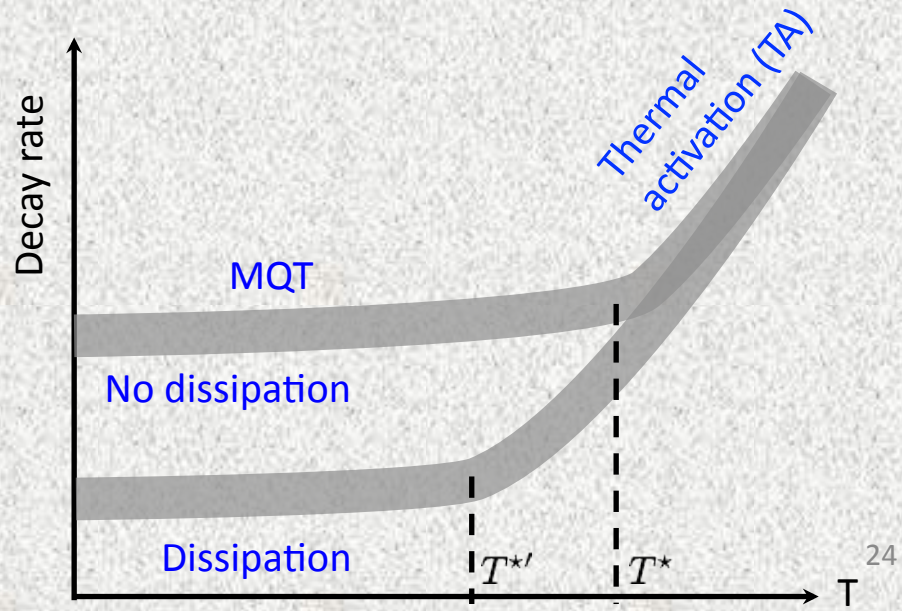
MGS-induced dissipation dominates



Application to Macroscopic Quantum Tunneling

Decay of persistent current state





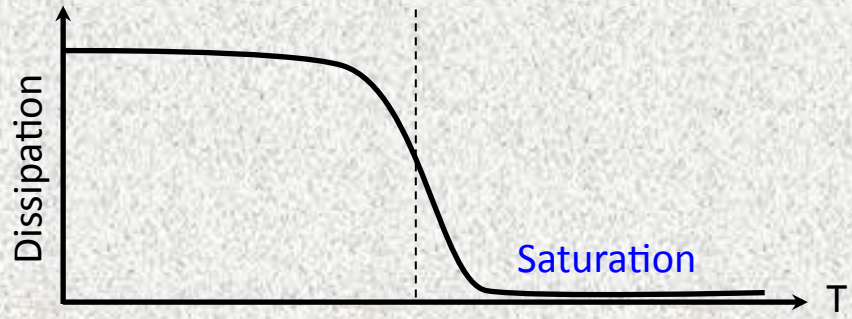
Temperature independent dissipation

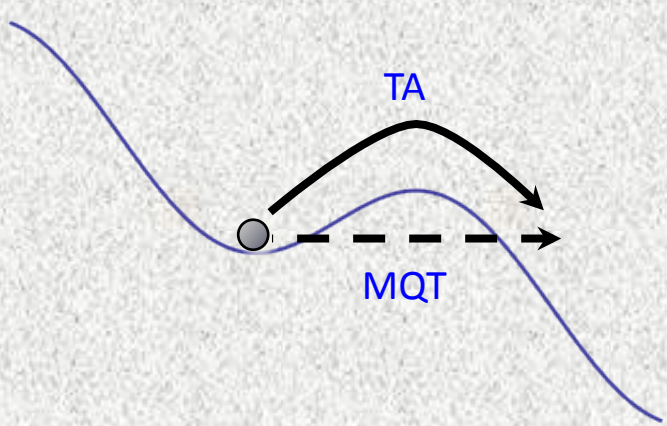
A. O. Caldeira and A. J. Leggett, PRL **54**, 2712 (1981)
H. Grabert and U. Weiss, PRL **53**, 1787 (1984)

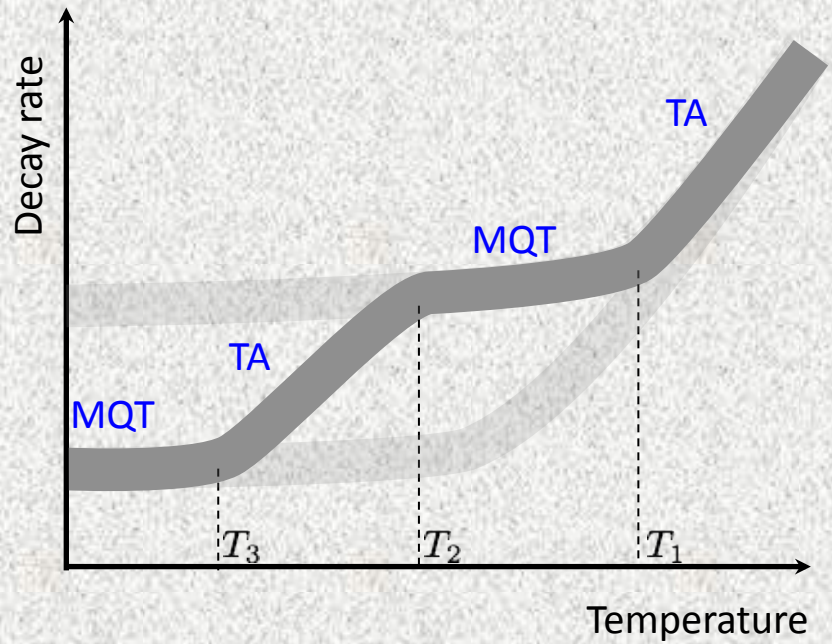
MGS induced dissipation – Reentrance Effect

Saturation with temperature

$$\gamma_0''(\omega, T) \sim \frac{1}{R_n C} \frac{\omega}{\varepsilon_m} \tanh\left(\frac{\varepsilon_m}{4k_B T}\right)$$

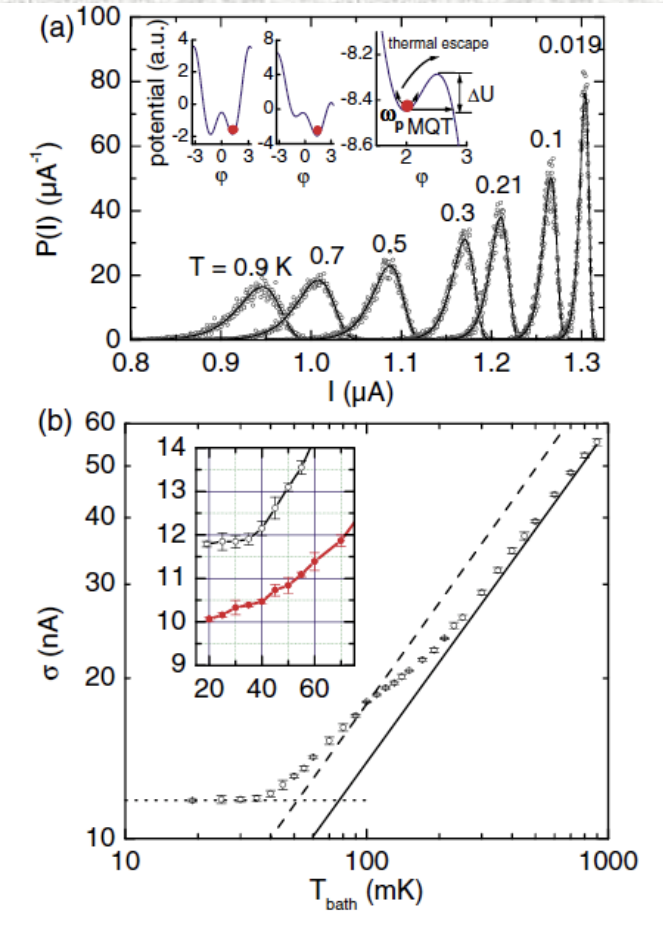




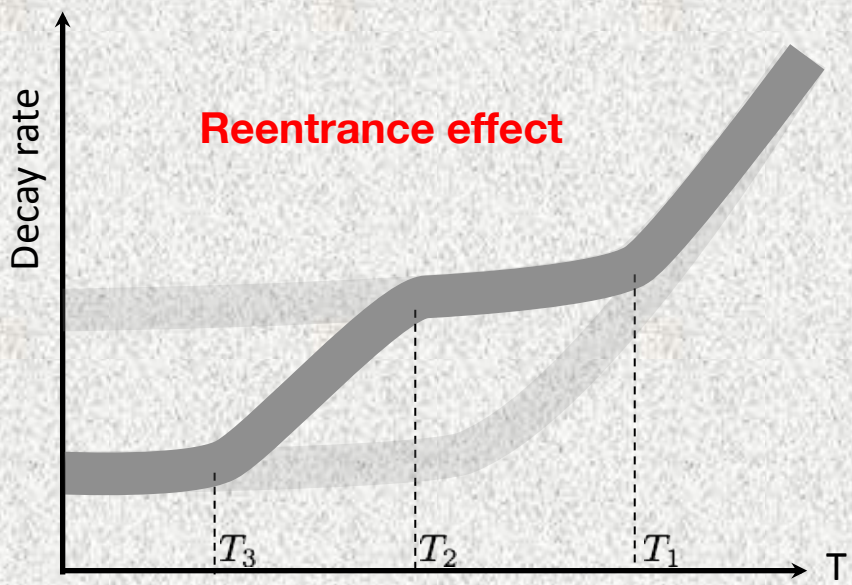


MGS induced dissipation – Reentrance Effect

YBCO Josephson junction



Bauch et al. PRL **94**, 087003 (2005)



MQT transition temperature

Grabert and Weiss, PRL **53**, 1787 (1984)

Euclidean action
for small fluctuation

$$S_{\text{eff}}[\varphi_b] + \sum_n \Lambda(\varphi_b, i\nu_n)\delta\varphi_n\delta\varphi_{-n}$$

$$\nu_n = 2\pi n T$$

Equation for transition
temperature

$$\Lambda(\varphi_b, i\nu_1) = 0$$

$$\Lambda(i\nu_n) = (C/8e^2)[\nu_n^2 - \omega_b^2 - i\nu_n\gamma_0(i\nu_n)]$$

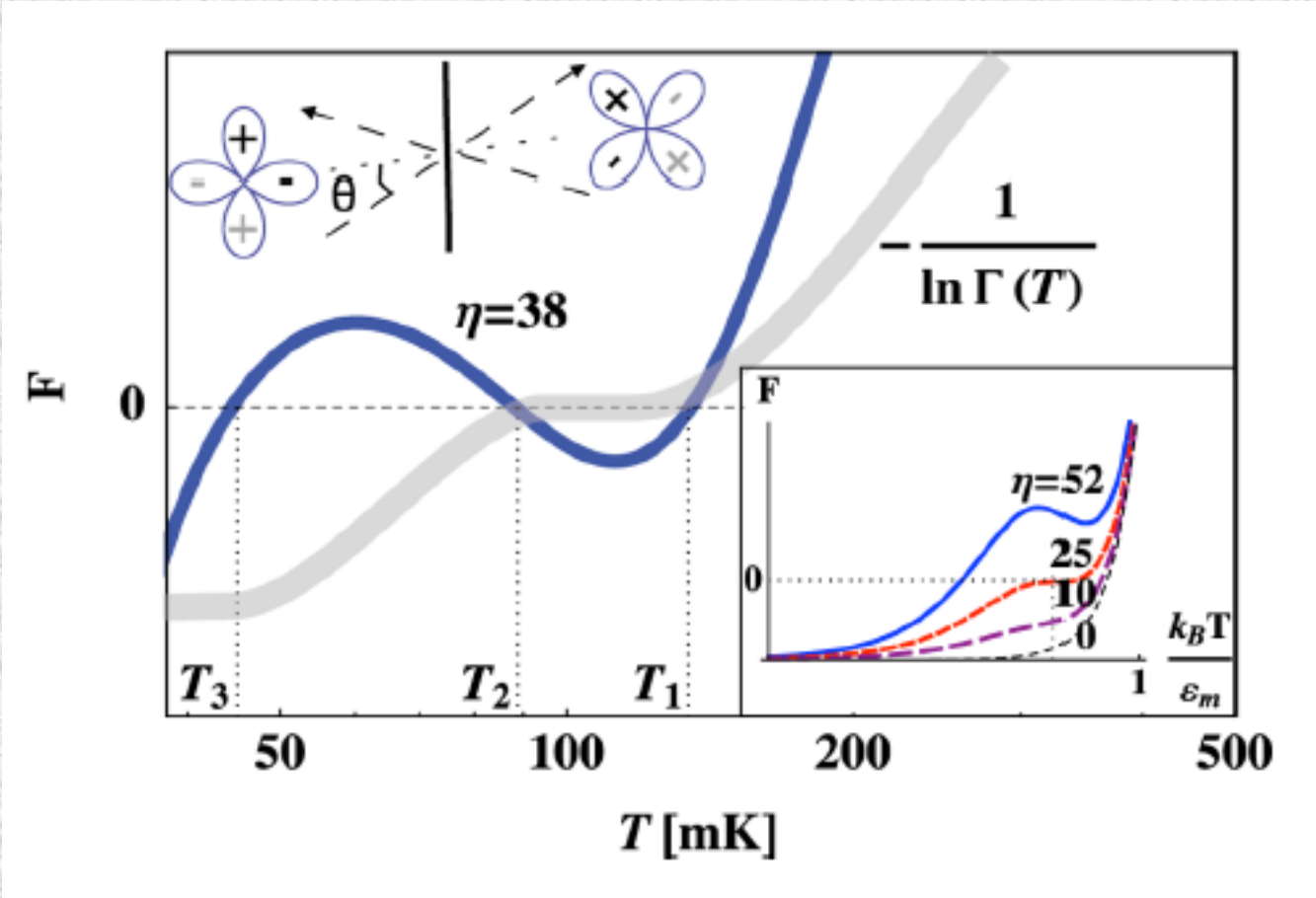
analytical continuation
of linear response

Model equation
(arbitrary misorientation)

$$(2\pi T)^2\left(1 + \frac{8\pi a}{R_n C}\int_0^{\varepsilon_m} d\varepsilon \frac{\tanh(\varepsilon/4T)}{\varepsilon^2 + (2\pi T)^2}\right) = \omega_b^2$$

coupling strength

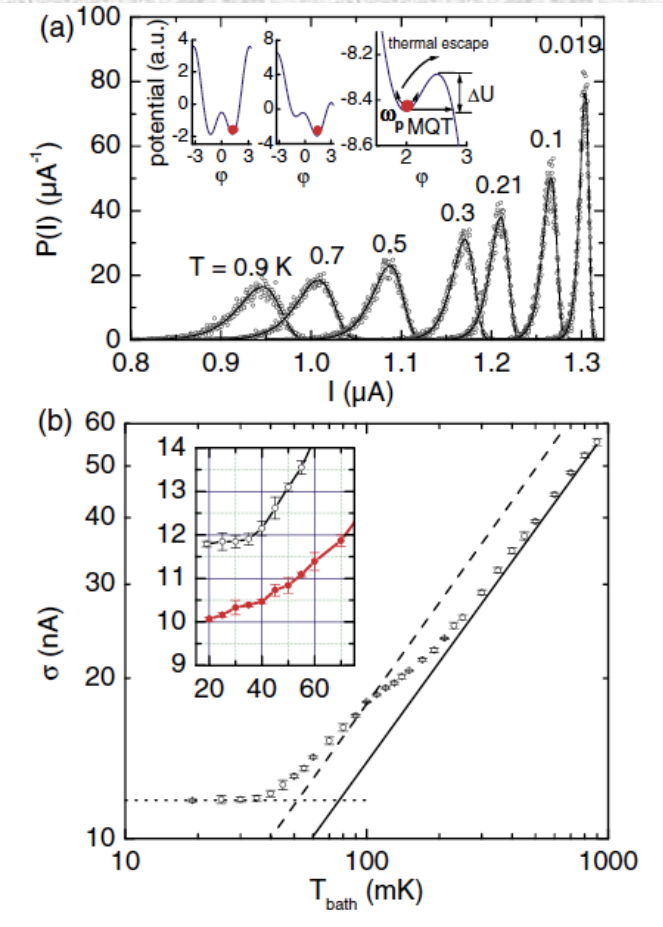
Graphic solution



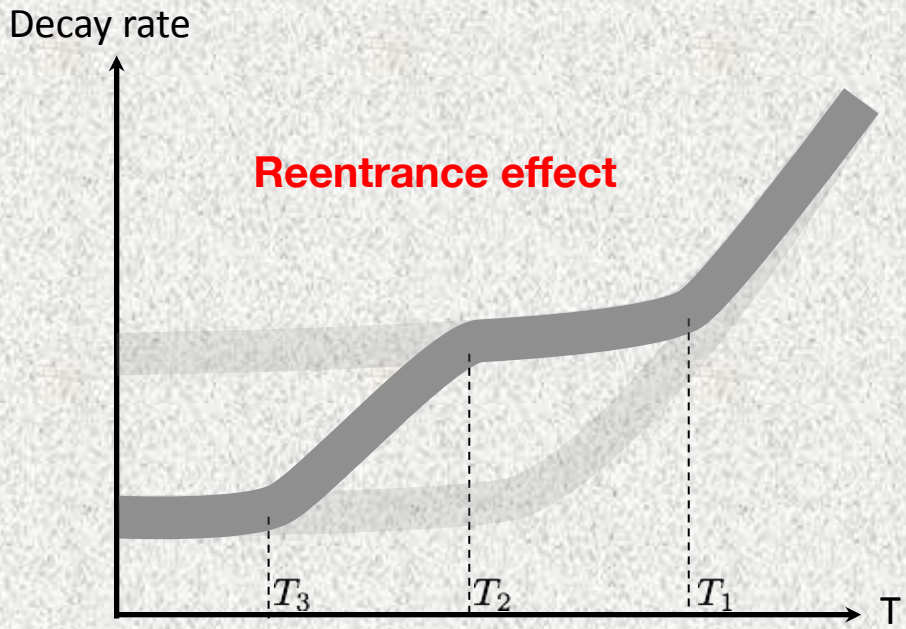
Critical temperatures are given by crossings of blue curve with line F=0

Comparison with experiment

YBCO Josephson junction



Bauch et al. PRL **94**, 087003 (2005)



Fitting parameters:

$\eta \approx 45; \quad \varepsilon_m \approx 320 \text{ mK}; \quad \hbar\omega_b \approx 1.7 \text{ K}$

Experimental input parameters:

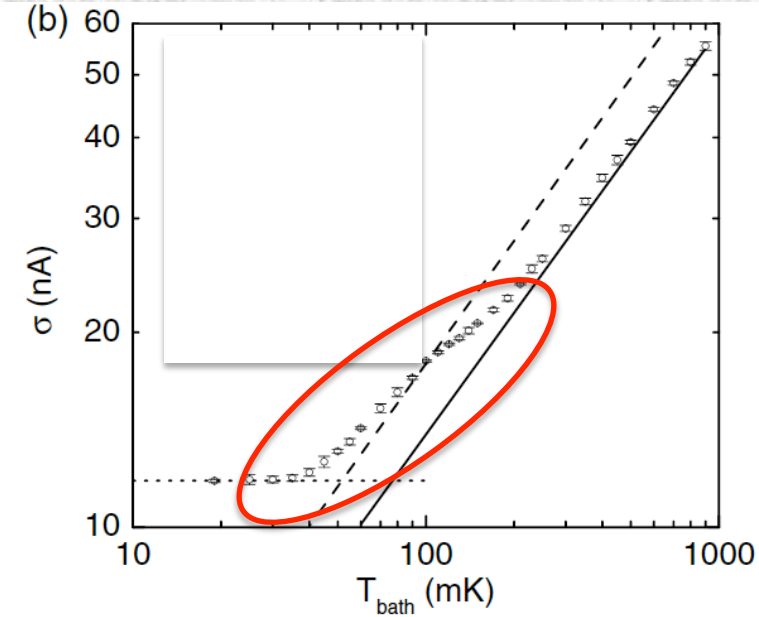
$I_c = 1.4\ \mu\text{A}; \qquad R_n = 500\ \Omega$

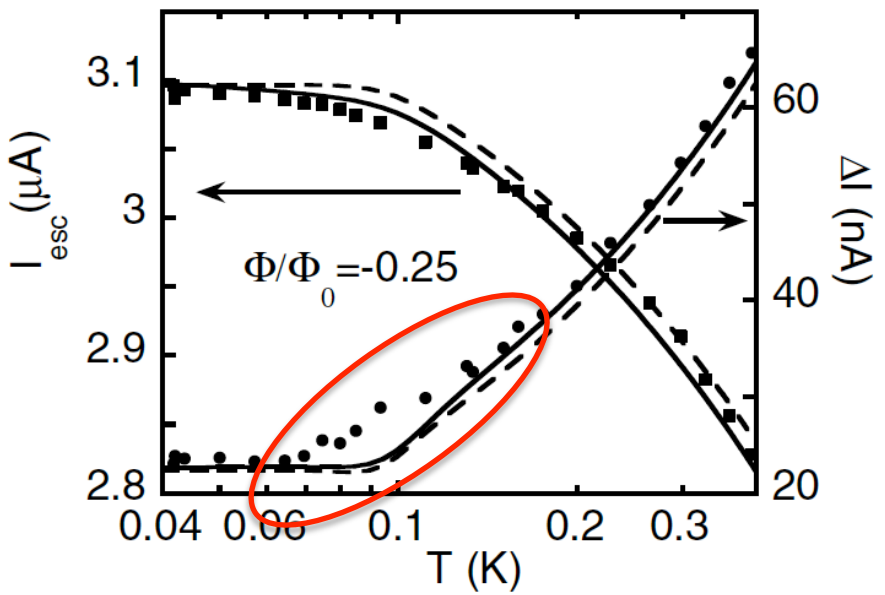
Extracted parameters:

$C_J \approx 36 \text{ fF}; \quad \Delta_0 \approx 16 \text{ K}$

J. Michelsen, VS, PRL, **105,** 127001 (2010)

Experiment





YBCO junction (d-wave)
Chalmers-Naples

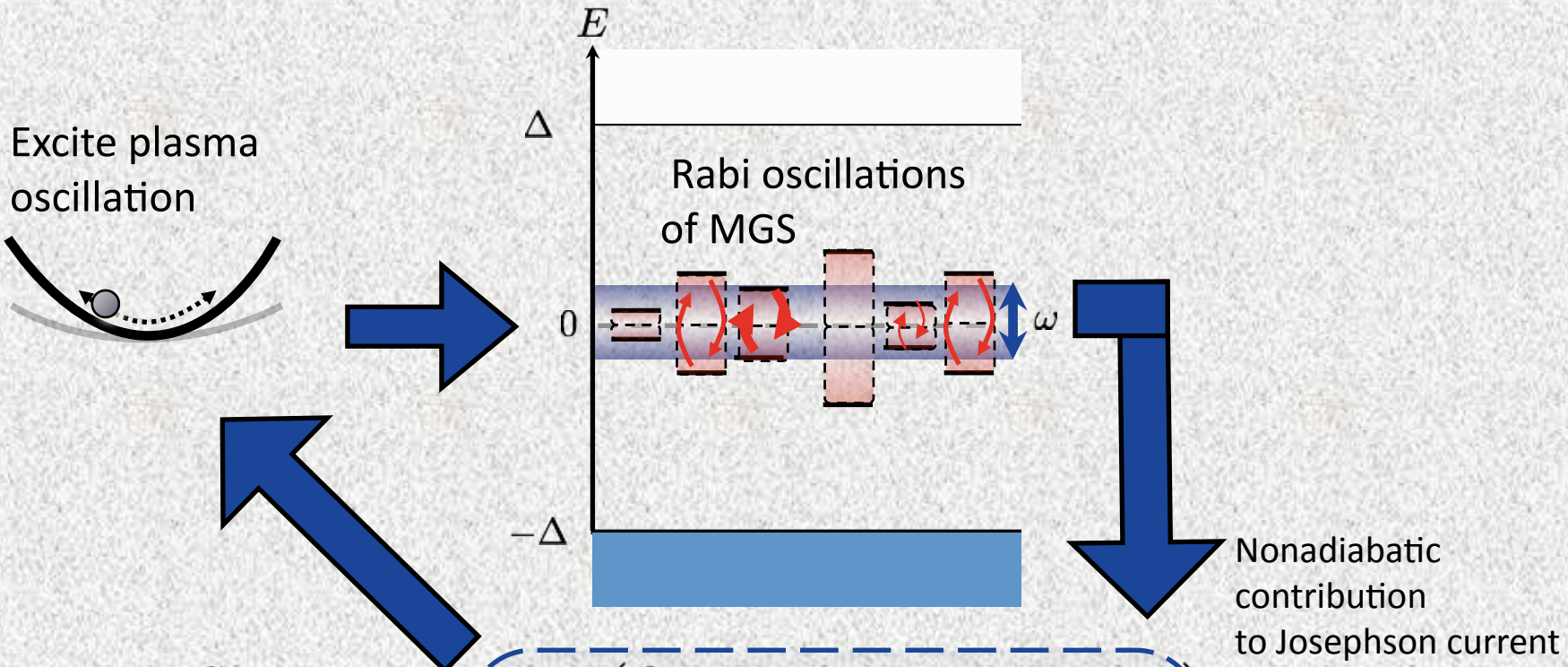
Bauch et al. PRL **94**, 087003 (2005)

Al-Al junction (s-wave)
Grenoble

Balestro et al. PRL **91**, 158301 (2003)

Real-time dynamics: nonlinear effects

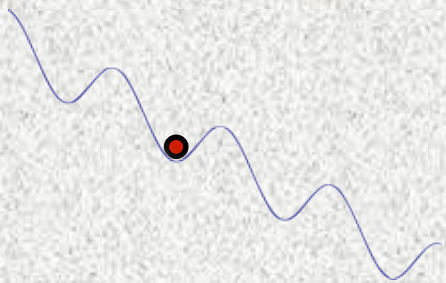
- **Phase drive preserves conducting modes** - transitions only occur between MGS of the same mode – MGS pairs
- **Linear response does not apply to spectroscopically sharp MGS** – driven far from equilibrium - nonlinear response despite small phase amplitude



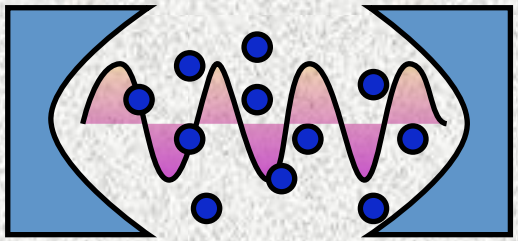
$$\frac{C}{2e}\ddot{\varphi} + I_J^{\mathrm{ad}}(\varphi) + 2e\sum_n \left(\frac{\partial \varepsilon}{\partial \varphi}(\rho_z - \rho_z^0) + \varepsilon \mathcal{A}\rho_+ + \varepsilon \mathcal{A}^* \rho_- \right) = I_{\mathrm{ext}}$$

New paradigm of Josephson dynamics

Conventional Josephson dynamics:
"Particle in a potential"



Josephson junctions with in-gap quasiparticles:
"Optical mode in medium of two level atoms"



- **Physics reminiscent of optical mode interacting with a medium of two-level atoms**

Non-linear resonance

$$\frac{C}{2e}\ddot{\varphi} + I_J^{\mathrm{ad}}(\varphi) + 2e\sum_n \left(\frac{\partial\varepsilon}{\partial\varphi}(\rho_z - \rho_z^0) + \varepsilon\mathcal{A}\rho_+ + \varepsilon\mathcal{A}^*\rho_-\right) = I_{\mathrm{ext}}$$

Slow varying amplitudes

$$\varphi(t) = \frac{1}{2}(\varphi_\omega(t)\mathrm{e}^{-i\omega t} + \mathrm{c.c.}) \qquad \rho_+(t) = \rho_\omega(t)e^{-i\omega t}$$

$$\varphi_\omega(t) = r(t)e^{i\vartheta(t)}$$

$$\dot{\rho}_\omega = -i(\varepsilon_0 - \omega - i\Gamma_2)\rho_\omega - i\omega\mathcal{A}_0\varphi_\omega\rho_z$$

$$\dot{\rho}_z = i\frac{\omega}{2}(\varphi_\omega\rho_\omega^* - \varphi_\omega^*\rho_\omega) - \Gamma_1(\rho_z - \rho_z^0)$$

Intrinsic broadening of MGS

Adiabatic evolution of MGS amplitudes

$$\partial_t\varphi \ll \Gamma_i\varphi$$

$$\rho_\omega = \frac{\omega\mathcal{A}_0\varphi_\omega}{\varepsilon_0 - \omega - i\Gamma_2}\rho_z$$

$$\rho_z = \rho_z^0 - \frac{(\omega\mathcal{A}_0 r)^2(\Gamma_2/\Gamma_1)}{(\varepsilon_0 - \omega)^2 + \Gamma_2^2 + (\omega\mathcal{A}_0 r)^2(\Gamma_2/\Gamma_1)}\rho_z^0$$

J. Michelsen, VS, Low Temp. Phys. **36**, 925 (2010)

Shortened equation for phase

$$-2i\dot{\varphi}_\omega + [-2\delta + \gamma(r)]\varphi_\omega = eI_e/\omega_p C$$

Nonlinear response of MGS

$$\gamma'(\omega, r) \approx \gamma_0' - \frac{\partial_\varphi^2 \bar{\varepsilon}_0 r^2}{\Gamma_1} \frac{\Gamma\gamma_0''}{\sqrt{(r\bar{\mathcal{A}}_0\omega)^2 + \Gamma^2}},$$

$$\gamma''(\omega, r) \approx \frac{\Gamma\gamma_0''}{\sqrt{(r\bar{\mathcal{A}}_0\omega)^2 + \Gamma^2}}.$$

Bifurcation curve

$$\delta = \frac{1}{2}\gamma'(r) \pm \frac{1}{2r}\sqrt{(eI_e/C\omega_p)^2 - [\gamma''(r)r]^2}$$

Maximum amplitude

$$r_m = \frac{\tilde{I}_e\Gamma}{\sqrt{\Gamma^2(\gamma_0'')^2 - \tilde{I}_e^2\bar{\mathcal{A}}_0^2\omega^2}}$$

$$\tilde{I}_e = (e/C\omega_p)I_e$$

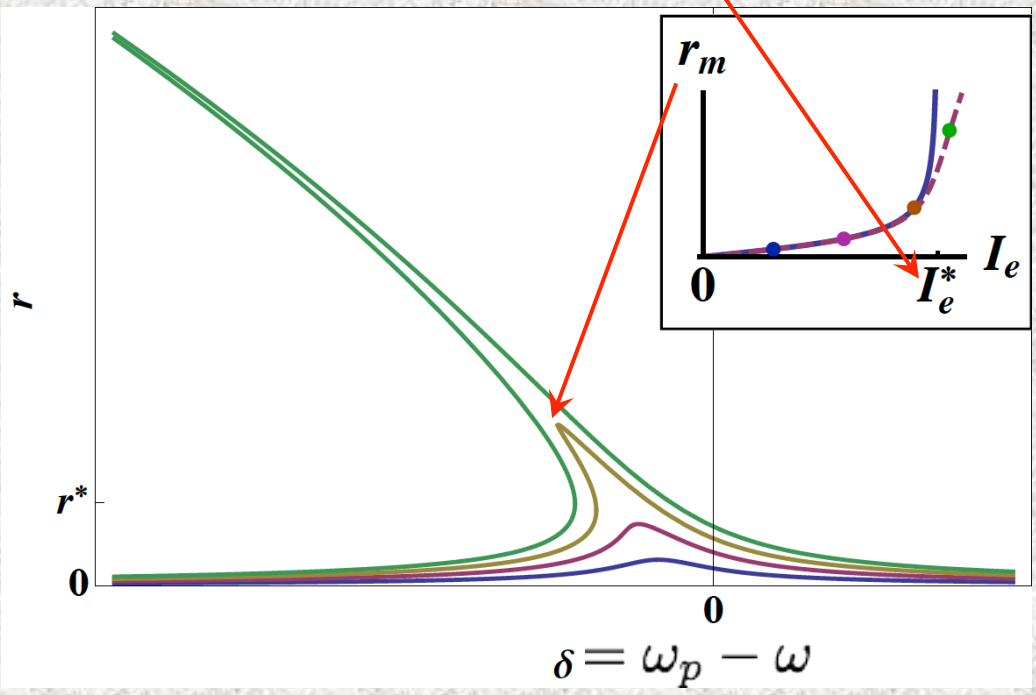
Phase response amplitude diverges at small drive

Nonlinear MGS response: nonlinear Josephson dynamics

$$I_{\text{ext}}(t) = I_e \cos(\omega t)$$

$$\frac{I_e^*}{I_c} = \frac{2\gamma_0''}{\omega_p}\frac{\Gamma}{\omega_p \mathcal{A}}$$

- Bifurcation regime **induced by MGS**
- Instability at finite current *I** due to MGS **saturation**



$$\begin{cases} \Omega(r) \ll \Gamma & \rho_z(r) = \rho_z^0, \text{ Linear response} \\ \Omega(r) \gg \Gamma & \rho_z(r) \sim \frac{1}{r}, \text{ Saturation} \end{cases}$$

$$\Omega(r) = \sqrt{\omega r \mathcal{A}}$$

Rabi frequency

amplitude of Josephson oscillation

$$\Gamma = \sqrt{\Gamma_1 \Gamma_2}$$

J. Michelsen, VS, Low Temp. Phys. **36**, 925 (2010)

Conclusions

- General theory of Josephson dynamics in junctions with low energy quasiparticles
- Josephson junctions with low energy Andreev bound states provide new paradigm of Josephson dynamics
- D-wave junctions give generic example
- Imaginary-time domain: Reentrance Effect in MQT
- Reentrance effect can be used for characterization of MGS
- Real-time domain: non-linear Josephson dynamics induced by MGS
- Interesting: time resolved experiments (echo, revival, solitons)